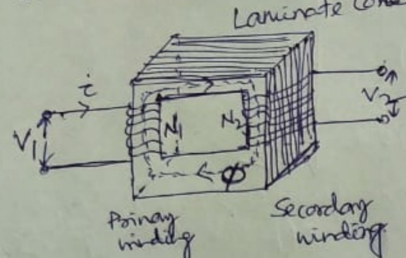
Elementary theory of Ideal Transformer

Ideal means no losses i.e no copper losses, no core losses & no magnetic leakage.

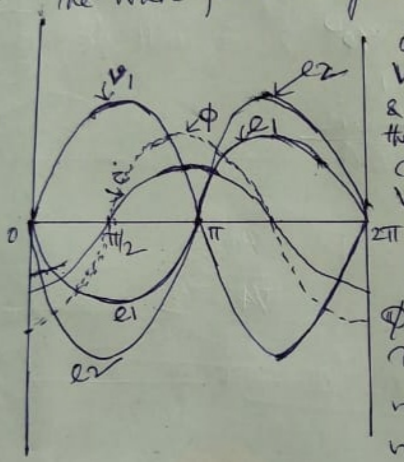
An Ideal Transformer consists of two purely inductive coils wound on a loss free core. But practically it does not exist. A practical transformer is realised through an ideal transformer step by step.



ϕ – mutual flux (flux linked both pry. & secy wdg)

Primary side is connected to a alternating voltage source of V_1 but secondary is kept open (no load connected).

The wave form diagram is given as



Explanation

When an a.c source of RMS value V_1 is connected to primary winding & since the coil is purely inductive ($R=0$) the primary side draws the magnetising current i (I_M) only. and lags behind V_1 by exactly 90^0 ($\pi/2$) and it's function is to magnetise the core & set up flux ϕ (proportional to i & No of turns). ϕ remains in same phase with i. This alternating flux 'ϕ' links with both primary & secondary winding & called mutual flux.

This alternating flux ϕ produces self induced EMF in primary winding & mutual induced EMF in the secondary winding according to faraday's Law of Electromagnetic induction.

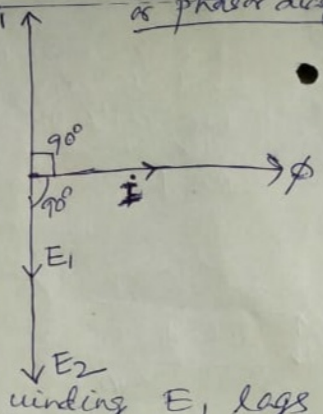
Self induced EMF $e_1 = -\frac{d\phi}{dt} \times N_1$ (e_1 lags ϕ by 90^0)

$$\left[\begin{aligned} \phi &= \phi_m \sin \omega t \\ -\frac{d\phi}{dt} &= -\phi_m N_1 \omega \cdot \cos \omega t = \omega N_1 \phi_m \sin(\pi/2 - \omega t) \\ &= \omega N_1 \phi_m \sin(\omega t - \pi/2) \end{aligned}\right]$$

the emf e_1 lags ϕ by $\pi/2$, & lags V_1 by π (180^0, antiphase)

Similarly ~~induced~~ emf e_2 is induced in secondary winding whose magnitude is proportional to secondary nos. of turns (N_2), $e_2 = -N_2 \frac{d\phi}{dt}$; It also lags ϕ by $\pi/2$ & in antiphase with V_1 (lags V_1 by 180^0)

The vector diagram is drawn as as phasor diagram



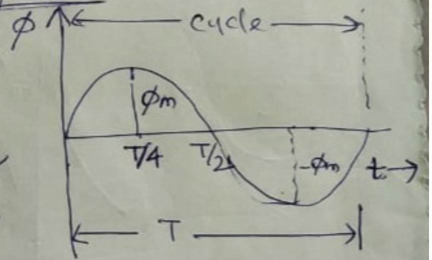
all values are in rms value

A.C voltage source of RMS value V_1 taken in y axis. So current I will be on x axis (exactly 90° lagging)
Then ϕ is in phase with I so ϕ will be on X axis.

Emf induced on primary winding E_1 lags ϕ by 90° (which is 180° opposite to V_1 also)

EMF induced on the secondary winding E_2 will also lag ϕ by 90° & it will be on the same axis as E_1

EMF Equation of Transformer



Eqn of flux $\phi = \phi_m \sin \omega t$

N_1 = Pry. no of turns

N_2 = Secy. "

ϕ_m = maximum flux $= B_m A$

B_m = maximum flux density

A = Area of Core

T = Time period = $1/f$

Emf induced per turn $= \frac{d\phi}{dt}$ or Average rate of change of flux -

at $t=0$, flux $\phi = 0$

at $t = T/4$ flux $\phi = \phi_m$

So change of flux $= \phi_m - 0 = \phi_m$

time interval $= T/4 - 0 = T/4$

So Avg. rate of change of flux $= \frac{\phi_m - 0}{T/4 - 0} = \frac{\phi_m}{T/4}$

$f = 1/T$ or $T = 1/f$

So Avg Emf per turn $= \frac{\phi_m}{1/4f} = 4f\phi_m$

RMS value of EMF $= 1.11 \times 4f\phi_m = 4.44 f \phi_m = 4.44 f B_m A$

The RMS value of Emf in primary winding

$E_1 = 4.44 f N_1 B_m A$, $E_2 = 4.44 f N_2 B_m A$

E_2 = EMF induced in the Secondary winding
$= 4.44 f N_2 B_m A$ or $4.44 f \phi N_2$

$E_1 = 4.44 f \phi N_1$

Dividing E_2 by E_1

$$\frac{E_2}{E_1} = \frac{4.44 f \phi N_2}{4.44 f \phi N_1} = \frac{N_2}{N_1} = K \text{ (a constant)}$$

K is called Transformation Ratio.

if $N_2 > N_1$ then $K > 1$ so step up Trf.

if $N_2 < N_1$ then $K < 1$ so step down Trf.

Since the transformer is Ideal & no load

$V_1 = E_1$, $V_2 = E_2$, V_2 = Terminal voltage of Secondary winding

& Input KVA = Output KVA

$$V_1 I_1 = V_2 I_2$$

or $\frac{V_2}{V_1} = \frac{I_1}{I_2}$ or $\boxed{\frac{E_2}{E_1} = \frac{I_1}{I_2} = K}$

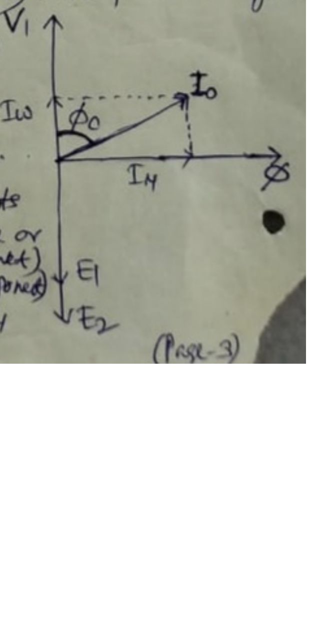
or $\boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$

Step-2 – Transformer with losses but no magnetic Leakage (Leakage Reactance)

(i) – Transformer on no Load condition –

Even if the transformer is on no Load, the primary input current is not wholly reactive, i.e I_0 does not lag behind V_1 by exactly 90° but lags behind V_1 by an angle $\phi_0 (< 90°)$ because of iron losses (hysteresis loss + eddy current losses) in the core.

The no load input power becomes $\boxed{W_0 = V_1 I_0 \cos\phi_0}$ (0 - subscript indicates no load condition). The phasor diagram is given as –



ϕ – mutual flux

I_0 – no load primary input current which lags behind V_1 by an angle ϕ_0.

V_1 vector is taken as reference on y axis.

power factor $= \cos\phi_0$

I_0 can be resolved into two components

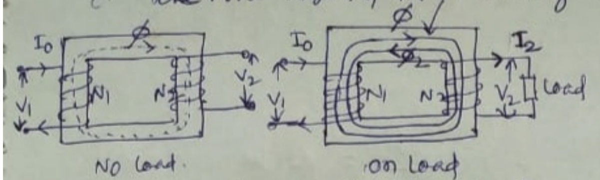
i.e along $V_1 \rightarrow I_0 \cos\phi_0$ or I_W (Active or working component)

& along $\phi \longrightarrow I_0 \sin\phi_0$ or I_M (magnetising component)

So that $I_0 = \sqrt{I_W^2 + I_M^2}$ or $I_0 = I_W + j I_M$

(ii) Transformer on loaded condition

∵ We have neglected the winding resistance & Reactances.



I_2 – refers to secondary current

I_2' – represents primary equivalent of secondary current I_2.

When the secondary is loaded, secondary current I_2 is drawn by the load. The magnitude & phase of the load current I_2 with respect to V_2 (secondary terminal voltage) depends upon the power factor of the load. Three types of load i,e Resistive (non-inductive), Inductive & Capacitive loads.

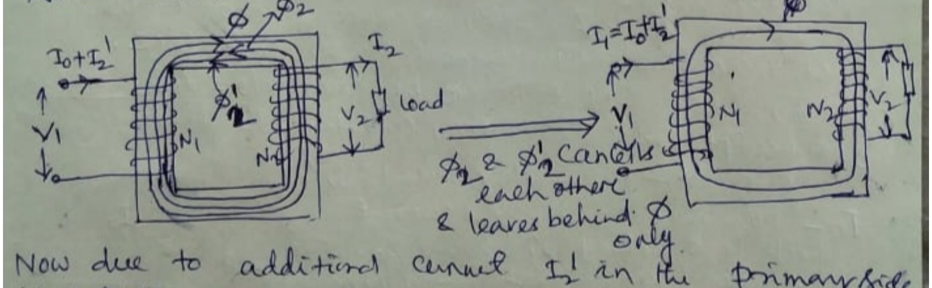
For Resistive load → I_2 is in phase with V_2

Inductive load → I_2 lags V_2

Capacitive load → I_2 leads V_2

The secondary current I_2 sets up its own Ampere Turns (N_2I_2) and hence its own flux ϕ_2 which is in opposition to main flux ϕ which is due to only I_0. The demagnetising secondary flux ϕ_2 weakens the main flux ϕ momentarily, hence primary induced Emf $E_1 \left(N_1\frac{d\phi}{dt}\right)$ tends to decrease. At the same time potential Difference across the primary winding ($V_1 - E_1$) tends to increase and hence more current flows in the primary winding. Let this additional current flows in the primary winding due to I_2 in the secondary i.e I_2' (Figure below is reffered) (I_2 - occurs in the secondary side but I_2' is on primary side) The additional primary current I_2' is also called as load component of primary current. This I_2' is antiphase with I_2 i,e 180° phase difference.

~~Now due to~~



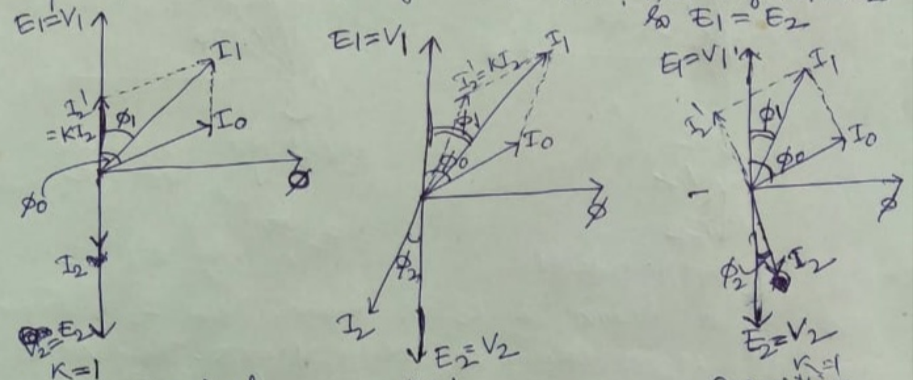
Now due to additional current I_2' in the primary side it sets up of own Ampere Turns ($= N_1I_2'$) & hence flux ϕ_2' which is in opposition to ϕ_2 (but along with ϕ) is equal in magnitude with ϕ_2.

$$\phi_2 = \phi_2' \text{ or } N_2I_2 = N_1I_2' \text{ or } I_2' = \frac{N_2}{N_1}I_2 = KI_2$$

The net core flux (ϕ) remains same practically from no load to full load. Hence core loss is practically constant under all load conditions.

I_1 is the vector sum of I_0 & I_2'.

The phasor diagrams for resistive, Inductive & capacitive load are given below. Assuming $N_1 = N_2$ so $E_1 = E_2$



$K=1$ Resistive load. $K=1$ Inductive load. $K=1$ Capacitive load.

ϕ_2 = Phase difference between Voltage & Current on secondary side

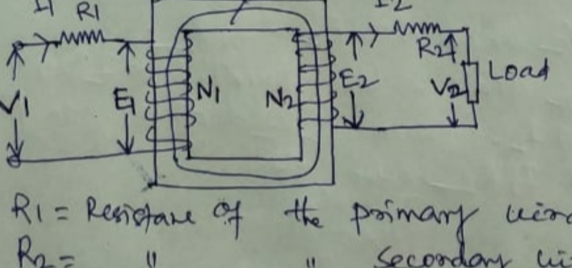
ϕ_1 = " " " " " primary side

ϕ_0 = Phase difference between " on primary side on no load

ϕ = Magnetising flux or main flux

I_2' vector is drawn exactly opposite to I_2 & magnitude depends on Transformation ratio K.
Then I_1 is drawn according to law of parallelogram of two vectors I_0 & I_2'. I_1 is the resultant vector of I_0 & I_2'.

Step-3 – Transformer with Winding Resistance but No magnetic leakage



$K = N_2/N_1$

I_1 = vector sum of I_0 & I_2'

$I_2' = K I_2$

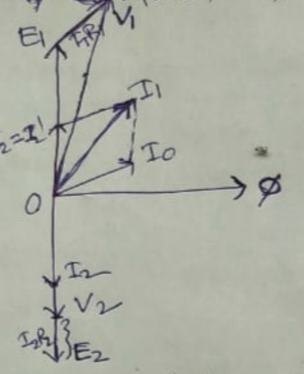
R_1 = Resistance of the primary winding

R_2 = " " secondary winding

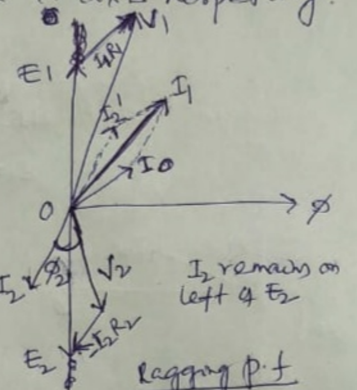
- $V_1 - I_1R_1 = E_1$ & $E_2 - I_2R_2 = V_2$

or $V_1 = E_1 + I_1R_1$

The vector diagrams for Unit power factor, Lagging p.f. & leading p.f is given below.
first E_1 & ϕ are drawn on Y & X axis respectively.



Unit power factor



Lagging p.f

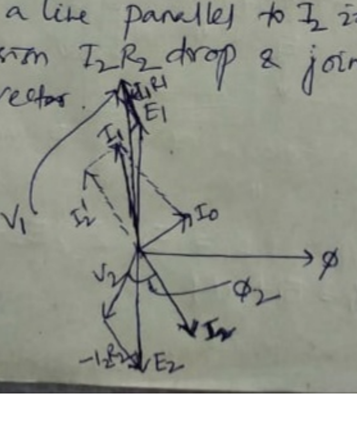
Then I_o is drawn & E_2 is drawn. Then I_2 is drawn. Then I_2' $(= KI_2)$ is drawn on positive Y axis. Then vector sum of I_2' & I_o is found out using law of parallelogram, which is the diagonal vector of I_2' & I_o.
Now $V_1 = E_1 + I_1R_1$, so voltage drop I_1R_1 is drawn parallel to I_1 at end of vector E_1 & join the end of I_1R_1 to O. Similarly $V_2 = E_2 - I_2R_2$. So V_2 is found out by substracting voltage drop I_2R_2. $(V_2 < E_2)$. from the end of E_2 substract I_2R_2.

for lagging p.f E_1, ϕ, E_2 are drawn
I_2 lags E_2, I_2' is drawn, then vector sum of I_2' & I_o is drawn as I_1. Then at the End of E_1, I_1R_1 voltage drop is drawn parallel to I_1. Then join the end of I_1R_1 to O which is vector V_1. $V_1 = E_1 + I_1R_1$

To find out V_2 which is $E_2 - I_2R_2$.
at the end of E_2 draw a line parallel to I_2 in reverse direction of I_2. Assign I_2R_2 drop & join from the center O to I_2R_2 vector.

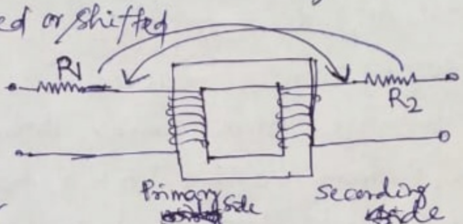
Leading. p.f

I_2 remains on right of E_2 (in leading)

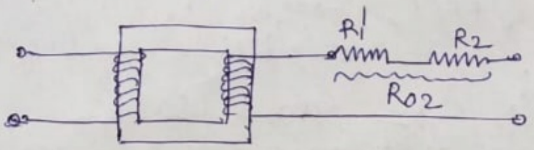


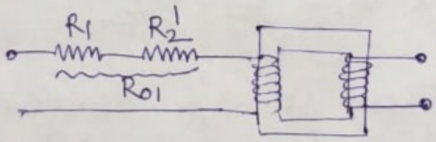
Equivalent Resistance (Referred to primary or Secondary)

The Resistance R_1 Can be transferred or shifted to the secondary side or the Resistance R_2 Can be transferred to the primary side.



The advantage of concentrating both the resistances in one winding is to make the calculations simple & easy.





Resistance R_1 ~~[illegible]~~ is transferred to secondary side as R_1' keeping in the principle that power loss in the primary side $(I_1^2R_1)$ must be equal to power loss in the secondary as $(I_2^2R_1')$.

So $I_1^2R_1 = I_2^2R_1'$

or $R_1' = R_1\left(\frac{I_1}{I_2}\right)^2 = R_1K^2$ (K = transformation Ratio or turn's ratio)

The total resistance as referred to secondary side $= R_2+R_1' = R_2+R_1K^2$

So total transformer resistance or effective Resistance as referred to secondary is (read as R zero two) R_{02}

$$\boxed{R_{02} = R_2+R_1' = R_2+K^2R_1}$$

Similarly Resistance R_2 is transferred to primary side as R_2' keeping in the principle that power loss in the secondary side $I_2^2R_2$ must be equal to the power loss in the primary as $I_1^2R_2'$.

$I_1^2R_2' = I_2^2R_2$

or $R_2' = R_2\left(\frac{I_2}{I_1}\right)^2 = \frac{R_2}{K^2}$

The total or effective resistance as referred to primary side is R_{01} (read as R zero one) $= \boxed{R_1 + R_2' = R_1+R_2/K^2}$

Remember → when R_1 transferred to secondary it becomes R_1K^2 or when R_2 transferred to primary it becomes R_2/K^2.

Magnetic Leakage In actual transformer all the flux linked with the primary winding donot wholly links with the secondary winding. But part of the flux (ϕ_{L1} or ϕ_{L2}) completes its magnetic path or circuit by passing through the air rather than the core. This flux is known as Leakage Flux (Primary leakage flux ϕ_{L1} or Secondary leakage flux ϕ_{L2})

Due to this leakage fluxes self induced emfs e_{L1} (in primary winding due to ϕ_{L1}) & e_{L2} (in secondary widing due to ϕ_{L2}) are induced.

The leakages fluxes accounts for leakage reactances $X_1 (= e_{L1}/I_1)$ & $X_2 (= e_{L2}/I_2)$ and known as primary & secondary leakage reactances respectively.

1. The leakage flux links only with one winding but not both.
2. The Primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1 so that $V_1 = I_1\sqrt{R_1^2 + X_1^2} = E_1$
or $V_1 = E_1 + I_1 Z_1$ (Z_1 = primary winding Impedance)

& $V_2 = E_2 - I_2 Z_2$ $\quad Z_1 = \sqrt{R_1^2 + X_1^2}$, $Z_2 = \sqrt{R_2^2 + X_2^2}$ Impedance of secondary winding

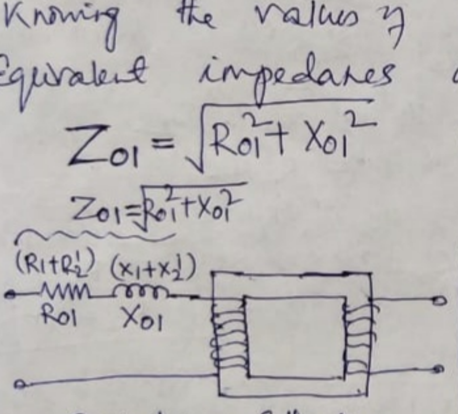
These leakage Reactances X_1 or X_2 can also be transfered to primary or secondary as in cases of Resistances.

$X_1' = X_1 K^2$ | $X_{01} = X_1 + X_2' = X_1 + X_2/K^2$

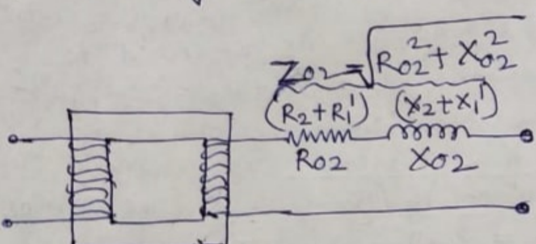
$X_2' = X_2/K^2$ | $X_{02} = X_2 + X_1' = X_2 + X_1 K^2$

Knowing the values of $R_{01}, X_{01}, R_{02}, X_{02}$ we can find Equivalent impedances as referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} \quad \& \quad Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$



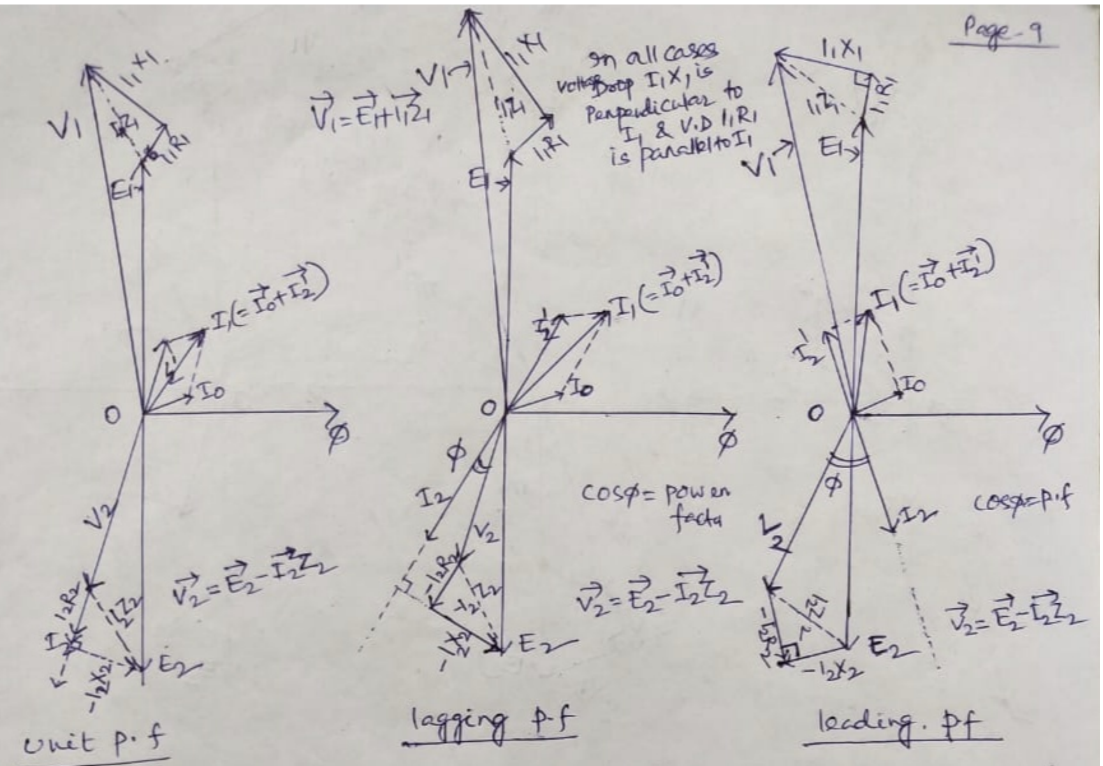
Equivalent or Effective Impedance of Trf. as reffenned to primary side



Equivalent or Effective Impedances of Trf. as referred to Secondary side

Step-3 Transformer with Resistances & Leakage Reactances

The vector diagrams or phasor representation of Transformer with different load conditions Resistive load (Unit p.f) Inductive load (Lagging p.f) & Capacitive load (leading p.f) are drawn.



Important Notes $\vec{V_2}$ must lag $\vec{E_2}$ in all cases & remains on the left side of $\vec{E_2}$. $\vec{V_1}$ always leads $\vec{E_1}$.

In case of unit power factor $\vec{I_2}$ & $\vec{V_2}$ remains on same line & phase difference between $\vec{E_2}$ & $\vec{I_2}$ must not be zero degree.

In case of lagging p.f $\vec{I_2}$ lags both $\vec{E_2}$ (greater angle) & $\vec{V_2}$. Voltage drop I_2Z_2 is deducted from E_2 to get V_2. $\cos\phi$ is the p.f of the load.

In case of leading p.f → $\vec{I_2}$ leads both $\vec{E_2}$ & $\vec{V_2}$. Voltage drop I_2Z_2 is deducted from E_2 to get V_2. $\cos\phi$ is the power factor of the load.

In all the cases voltage drop I_2R_2 is parallel to I_2 & voltage drop I_2X_2 is perpendicular to I_2

Drawing procedure → first of all draw a vertical line along Y axis with centre at 'O'. Then line on X' axis to represent flux vector (ϕ). Then draw the no load current I_0 (usually very small) & it is 1 to 2% of full load current. Then draw the secondary load current $\vec{I_2}$ with reference to $\vec{E_2}$. It will be to the left of $\vec{E_2}$ in case of unit & lagging p.f but to the right of $\vec{E_2}$ if leading p.f. Then draw I_2' (exactly opposite to $\vec{I_2}$) in reverse direction. Then draw the resultant of $\vec{I_0}$ & $\vec{I_2'}$ using law of parallelogram.

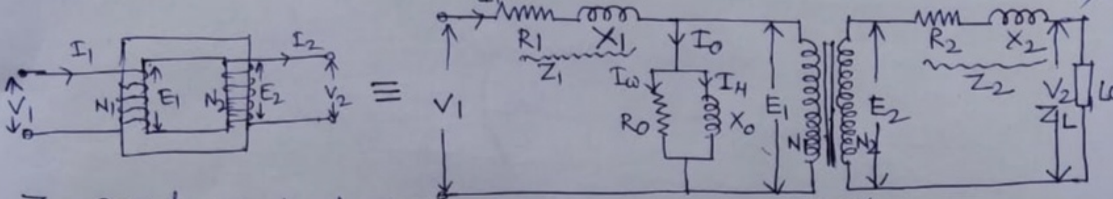
vector I_1 will be the diagonal of parallelogram..
Then at the end of vector $\vec{E_1}$ draw voltage drop I_1R_1 parallel to I_1
& I_1X_1 perpendicular to I_1R_1. $I_1Z_1 = I_1R_1 + jI_1X_1$ or $I_1\sqrt{R_1^2 + X_1^2}$
$\vec{V_1} = \vec{E_1} + \vec{I_1Z_1}$ & $\vec{V_1}$ is found out.

To find out V_2 $\quad \vec{V_2} = \vec{E_2} - \vec{I_2Z_2}$

first draw perpendicular to $\vec{I_2}$ at the End of $\vec{E_2}$. Then assign I_2X_2 drop. Then assign I_2R_2 drop along or parallel to I_2 in a reverse direction. At the end of I_2R_2 drop join the point 'O' to get $\vec{V_2}$.

Explain Equivalent Circuit - The Transformer can be resolved into an equivalent circuit in which the resistance & leakage reactance of the transformer are imagined to be external to the winding whose only function is to transform the voltage (from E_1 to E_2). The no load current 'I_0' is simulated by pure inductance 'X_0' taking the magnetising component 'I_M' and a non inductive resistance resistance 'R_0' taking the active or working component 'I_ω' connected in parallel across the primary circuit. 'I_ω' is in parallel with 'I_M'

& $\vec{I_0} = \vec{I_\omega} + \vec{I_M}$ & $I_1 = \vec{I_0} + \vec{I_2'}$ (I_2' = Primary equivalent of secondary current I_2)



Ideal Transformer (Figure-2)

Z_L = Impedance of load at secondary side

V_2 = Terminal voltage across the load

$E_2/E_1 = N_2/N_1 = K$ (Transformation Ratio or Turn's ratio)

To make the transformer calculations simpler, it is preferred to transfer voltage, current & impedance either to primary or the secondary side. In this calculations will be concentrated in one winding only.

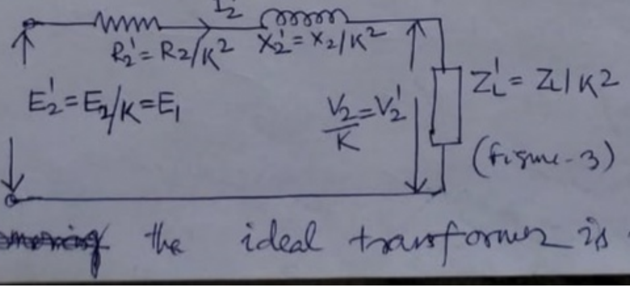
Primary equivalent of the secondary induced voltage (E_2) is termed as $E_2' = E_2/K = E_1$

Similarly, Primary equivalent of secondary terminal voltage (V_2) or output voltage is $V_2' = V_2/K$

Primary equivalent of secondary current is $I_2' = KI_2$

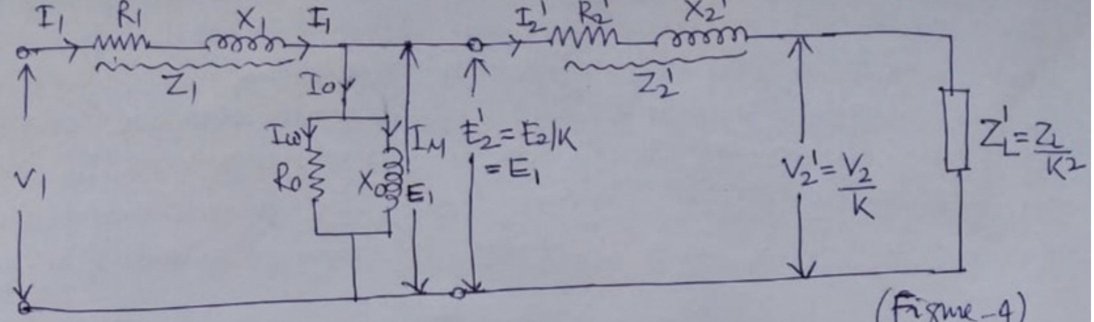
$R_2' = R_2/K^2$, $X_2' = X_2/K^2$, $Z_2' = Z_2/K^2$, $Z_L' = Z_L/K^2$

The secondary side of the figure-2 is transferred to primary side and redrawn as



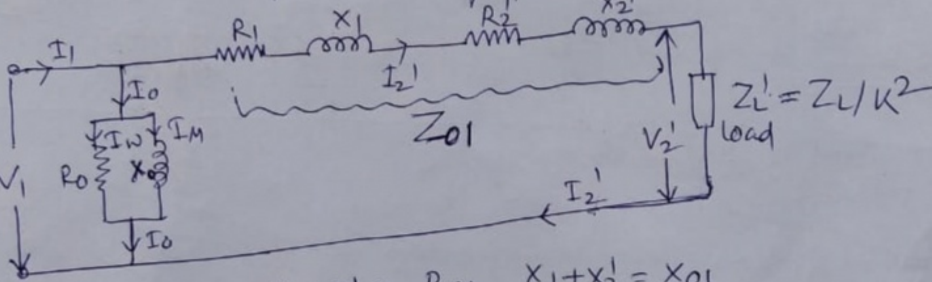
(figure-3)

& ~~removing~~ the ideal transformer is removed. This Circuit diagram is added with the primary side circuit (of Figure-2) to obtain the exact equivalent circuit.



(Figure-4)

which can be further simplified as



$$R_1 + R_2' = R_{01}, \quad X_1 + X_2' = X_{01}$$

$$\& \quad Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

Always equivalent circuit of figure-4) is referred.

~~Types of Losses in Transformer~~ –

Regulation of Transformer – Regulation of transformer is defined as the change in secondary terminal voltage from no load to full load and expressed as a percentage of no load or full load secondary voltage. If it is expressed as a percentage of no load secondary terminal voltage then it is called regulation 'down'. Again if it is expressed as a percentage of full load secondary terminal voltage it is called regulation 'up'. Primary voltage was kept to be constant.

$$\%\text{age regulation down} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$$

$$\%\text{age regulation up} = \frac{{}_0V_2 - V_2}{V_2} \times 100$$

${}_0V_2$ = no load secondary terminal voltage

V_2 = terminal voltage at full load

Different types of Losses in a Transformer

Two types of losses occur in a transformer
(i) Core loss or Iron loss - It includes both hysteresis loss and eddy current loss. Core loss remains practically constant at all loads. Hysteresis loss is given by the formulae $W_h = \eta B_{max}^{1.6} f V$ watt

where - B_{max} = maximum flux density
f = frequency of supply
V = Volume of core in m^3
η = Steinmetz hysteresis Coefficient

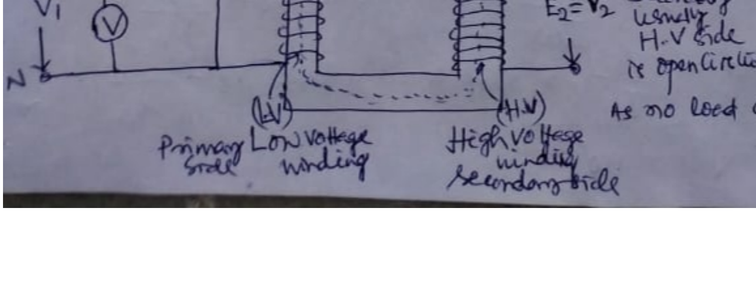
& Eddy current is given by the formulae
$W_e = K \cdot B_{max}^2 f^2 t^2 V^2$ watt
where K is Constant, t = thickness of lamination, B_{max} = maximum flux density, f = frequency & V = volume of core

Core loss can be found out by Open Circuit Test of transformer.

(ii) Copper loss - This loss is due to the ohmic resistance of transformer windings. This can be found out by Short circuit test.

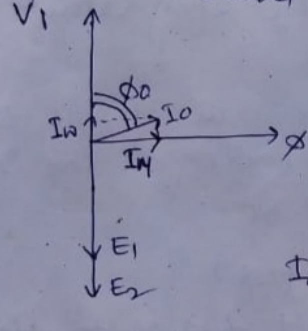
Open Circuit test (or No Load test)

The main purpose of open circuit test of transformer is to determine core loss or no load loss & no load current I_0 which is helpful in finding out no load parameters such as core loss conductance (G_0) or core loss resistance R_0, magnetising reactance (X_0) or Susceptance (B_0)



W - Wattmeter
C.C - Current coil of watt meter
P.C - Pressure coil of " "
A - Ammeter of (Low range)
As no load current is usually 2% of Full load current
V_1 = Input voltage
V - Volt meter

One winding of the transformer usually H.V winding is left open circuited because core loss depends on B_{max} or ϕ_m(mutual flux) and strength of mutual flux depends on magnitude of voltage.
L.V winding is connected to a supply of rated voltage & frequency (magnitude of voltage & frequency as printed in the name plate)
A wattmeter, Voltmeter & an ammeter are connected in low voltage winding. As rated voltage is applied to the L.V winding normal flux will be setup in the core & normal iron loss will occur which is recorded by the wattmeter(W).
The primary no load current I_0 measured by Ammeter (A) is very small (2% of Full load current). Copper loss is negligibly small in primary side & nil in secondary side. Hence the wattmeter reading practically represents the core loss under no load condition. Core loss also remains practically constant from no load to full load. The noload vector diagram is given as



The no load current I_0 lags V_1 by ϕ_0. I_0 can be resolved into two components.

$I_0 \cos\phi_0 \rightarrow$ Active component = I_w (or Wattfull component) along V_1

$I_0 \sin\phi_0 \rightarrow$ magnetising component = I_μ (or Watt less component) along ϕ

The reading of Wattmeter can be written as

$$W = V_1 I_0 \cos\phi_0$$

or $$\cos\phi_0 = \frac{W}{V_1 I_0}$$

Reading of W, V_1, & I_0 can be recorded from Wattmeter, voltmeter & Ammeter

Active component $I_w = I_0 \cos\phi_0$ is known

magnetising component $I_\mu = I_0 \sin\phi_0$ also known

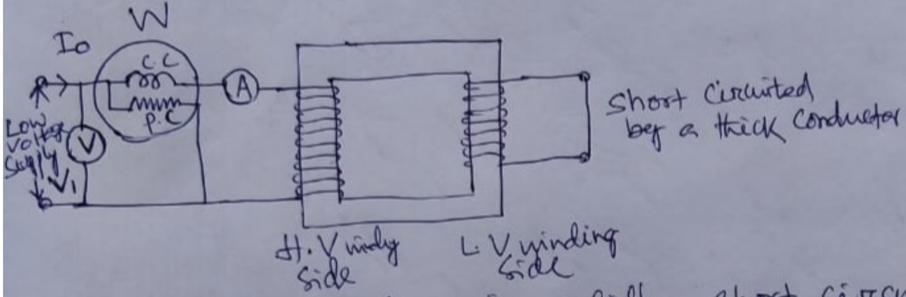
Now core loss resistance $R_0 = \frac{V_1}{I_w}$ & conductance $G_0 = \frac{I_w}{V_1}$

magnetising Reactance $X_0 = \frac{V_1}{I_\mu}$, admittance $Y_0 = \frac{1}{X_0}$

& Susceptance $B_0 = \sqrt{Y_0^2 - G_0^2}$

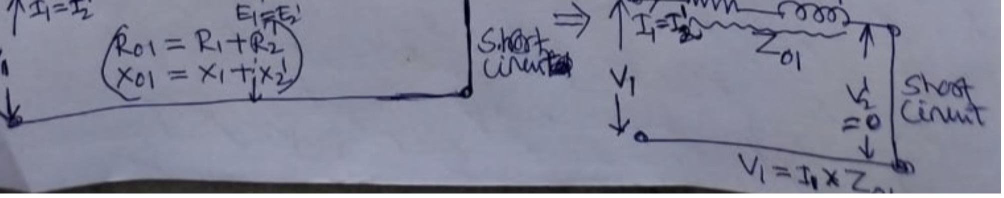
Short Circuit Test (or Impedance Test)

The purpose of this test is to find out (i) Copper loss at full load (and at any desired load) & hence efficiency of the transformer, (ii) - Equivalent impedance (Z_{01} or Z_{02}), leakage reactance (X_{01} or X_{02}) & total resistance of the transformer as referred to the winding in which measurements are done, (iii) - total voltage drop in the transformer as referred to primary or secondary.



Low voltage winding is solidly short circuited by a thick conductor or through an ammeter of very very low resistance to indicate the rated load current.
Usually 5% of the rated primary voltage is applied through a Variac (Auto transformer) & is continuously increased till the full load currents as indicated by the respective ammeters.

Since the applied voltage is a very small percentage of rated voltage the mutual flux produced is also very small & hence core loss is very very small & hence neglected. Hence the wattmeter reading practically represents the copper loss at full load for the whole transformer ie, primary + secondary copper losses, $I_1^2 R_{01}$ or $I_2^2 R_{02}$. The equivalent circuit under short circuit condition is given below as



In the circuit diagram the no load current component I_0 being very very small it is neglected. So $I_1 = I_2'$

V_1 = voltage required to circulate the rated load current $= (V_{SC})$

then $V_1 = I_1 Z_{01}$ reading V_1 can be recorded, reading of I_1 also recorded.

$$Z_{01} = \frac{V_1}{I_1}$$

Also reading of watter is recorded which is the copper losses of whole transformer ($I_1^2 R_{01}$).

$$So \quad W = I_1^2 R_{01}$$

$$or \quad R_{01} = \frac{W}{I_1^2}$$

$$Hence \; X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{\left(\frac{V_1}{I_1}\right)^2 - \left(\frac{W}{I_1^2}\right)^2}$$

V_1 = Impedance drop of the two windings

R_1 = Resistance of primary winding can be measured with the help of multimeter (or low voltage D.C supply along with D.C voltmeter & Ammeter connected to the transformer) R_1 can be found out. So R_2' (primary equivalent resistance of secondary winding) is found out

as $R_2' = R_{01} - R_1$ [Power input on short circuit $V_1 I_1 \cos\phi_{sc}$ = copper loss $= I_1^2 R_{01}$, or $\cos\phi_{sc} = \frac{I_1 R_{01}}{V_1}$]

Example → A single phase transformer has a turns ratio of 6. Resistance of the primary winding are 0.9Ω & 5Ω respectively & those of the secondary are 0.03Ω & 0.13Ω respectively. If 330V at 50Hz be applied to the H.V winding with the L.V winding short circuited, find the current in the L.V winding & it's power factor. Neglect magnetising current.

Solution Turn ratio or Transformation ratio is given as 6 which means 6:1, that implies if primary = 06 no. of turns secondary no of turns = 01. So $K = 01/06 = 1/6$, $K^2 = 1/36$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 0.9 + 0.03 \times 36 = \underline{1.98\,\Omega}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 5 + 0.13 \times 36 = \underline{9.68\,\Omega}$$

$$Z_{01} = \sqrt{9.68^2 + 1.98^2} = \underline{9.9\,\Omega}, \quad V_{SC} = V_1 = 330\,V \text{ (given)}$$

F.L primary current $I_1 = V_1/Z_{01} = 330/9.9\,\Omega = \underline{100/3\,A} = 33.33\,A$

As I_0 is negligible $I_1 = I_2'$ or $I_2' = 100/3\,A$ or $KI_2 = 100/3\,A$ or $I_2 = \frac{100/3}{1/6} = 200$ Amp

∴ full load ~~secy. current~~ secondary current = 200 Amp, Power factor $\cos\phi_{sc} = \frac{I_1 R_{01}}{V_1} = \frac{100 \times 1.98}{3 \times 330}$ = 0.2 lagging

Efficiency of Transformer - Also called ordinary efficiency or Commercial efficiency. The efficiency of a transformer at a particular load & power factor is defined as the ratio of output power to input power in Watts or Killowatts.

$$\text{Efficiency } (\eta) = \frac{\text{Output (in Watt or KW)}}{\text{Input (in Watt or KW)}} \text{ or } \frac{\text{Input} - \text{losses}}{\text{Input}}$$

$$\text{or } \frac{\text{output}}{\text{output} + \text{Losses}} = \frac{\text{Output}}{\text{Output} + \text{Copper losses} + \text{Iron losses}}$$

$$\eta = \frac{\text{output}}{(\text{output} + \text{Copper loss} + \text{Iron loss})} \times 100$$

Efficiency at Different loads :- Since copper loss is directly proportional to the Square of load current, it varies at different loads & hence efficiency will be different.

If full load copper loss $= I^2R$ where I = full load current

at half load i.e at $I/2$, loss will be $\left(\frac{I}{2}\right)^2 R = I^2R/4$

which is $\frac{1}{4}$th of full load copper loss.

Similarly at 3/4th load copper loss will be $\left(\frac{3I}{4}\right)^2 \times R$

$= \frac{9}{16}(I^2R)$ is $\frac{9}{16}$th part of full load copper loss

& efficiency will be changed at different loads.

Efficiency & power factor - If 'I' is the full load current & $\cos\phi$ is p.f of the load, active component $I\cos\phi$ must be constant. In this context if power factor changes (depends on type of load) i.e low or high, then full load current increases or decreases, for which copper loss increases or decreases thereby affecting the value of efficiency.

Condition for Maximum Efficiency :-

$$\eta = \frac{\text{Input} - \text{losses}}{\text{input}}$$, Copper loss $(W_c) = I_1^2 R_{01}$ or $I_2^2 R_{02}$

Iron loss or core loss $= W_i$

Considering from the primary side

Primary Input $= V_1 I_1 \cos\phi_1$

Output $=$ Input $-$ losses $= V_1 I_1 \cos\phi_1 - (W_c + W_i)$

efficiency $\eta' = \frac{\text{output}}{\text{Input}} = \frac{V_1 I_1 \cos\phi_1 - W_c - W_i}{V_1 I_1 \cos\phi_1}$

or $\eta' = \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1}$

$= 1 - \frac{I_1^2 R_{01}}{V_1 I_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$

efficiency will be maximum when $\frac{d(\eta)}{dI_1} = 0$

or $\frac{d}{dI_1}\left(1 - \frac{I_1^{\cancel{2}} R_{01}}{V_1 \cancel{I_1} \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}\right) = 0$

or $0 - \frac{R_{01}}{V_1 \cos\phi_1}\frac{d(I_1)}{dI_1} - \frac{W_i}{V_1 \cos\phi_1}\frac{d(1/I_1)}{dI_1} = 0$

or $-\frac{R_{01}}{V_1 \cos\phi_1} \times 1 - \frac{W_i}{V_1 \cos\phi_1} \times \left(\frac{-1}{I_1^2}\right) = 0$

or $-\frac{R_{01}}{V_1 \cos\phi_1} + \frac{W_i}{V_1 \cos\phi_1} \cdot \frac{1}{I_1^2} = 0$

or $\frac{W_i}{\cancel{V_1 \cos\phi_1}} \cdot \frac{1}{I_1^2} = \frac{R_{01}}{\cancel{V_1 \cos\phi_1}}$

or $W_i = I_1^2 R_{01} = W_c$

i.e. $\boxed{\text{Iron loss} = \text{copper loss}}$ which is the condition for maximum efficiency

Load corresponding to maximum efficiency:-

$W_i = W_c = I_1^2 R_{01} = I_2^2 R_{02}$ where I_2 = Secondary load current

$\Rightarrow I_2^2 = \frac{W_i}{R_{02}}$ or $I_2 = \sqrt{\frac{W_i}{R_{02}}}$

All day efficiency:- All day efficiency of transformer is defined as the ratio of output energy (kWh) to the ratio of Input energy (kWh) for 24 hours.

Example - Find the all day efficiency of 500 KVA distribution transformer whose copper loss & Iron loss at Full load are 4.5 KW and 3.5 KW respectively. During 24 hrs. if is loaded as under:

No of hours	Loading in KW	power factor
6	400	0.8
10	300	0.75
4	100	0.8
4	0	–

Solution:- $\text{efficiency}_{\text{allday}} = \dfrac{\text{output (KWh)}}{\text{Output + Losses}}$

Transformer output in 24 hrs. $= 400\times6 + 300\times10 + 100\times4 + 0$
$= 2400 + 3000 + 400 = 5800$ KWh

Since the transformer operates at different power factors for different time intervals the load current changes with p.f & accordingly copper loss changes. But Iron loss remains constant.

For 24 hours Iron loss $= 24 \times 3.5$ KWh $= 84$ kWh

Copper loss:- Since copper loss is proportional to I^2 or $(KVA)^2$

400 KW at 0.8 p.f $= \dfrac{400}{0.8} = 500$ KVA

300 KW at 0.75 pf $= \dfrac{300}{0.75} = 400$ KVA

100 KW at 0.8 pf $= \dfrac{100}{0.8} = 125$ KVA

Copper loss at F.L of 500 KVA = 4.5 KW (given)

So Copper loss at 400 KVA $= 4.5 \times \left(\dfrac{400}{500}\right)^2 = 2.88$ KW

Copper loss at 125 KVA $= 4.5 \times \left(\dfrac{125}{500}\right)^2 = 0.281$ KW

Total Copper loss in 24 hrs $= 6\times4.5 + 10\times2.88 + 4\times0.281 + 4\times0$
$= 56.924$ kWh

Total loss in 24 hrs $= 84 + 56.924 = 140.924$ kWh

$$\eta_{allday} = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{5800}{5800 + 140.924} = 97.6\%$$

Parallel operation of Singlephase Transformer

Necessity – (1) increased load demand
(2) Continuity in Service or reliability
(3) maintenance & repair
(4) increased efficiency

Certain definite Conditions must be Satisfied before parallel operation.

(1) Primary windings of transformers must be Connected to Same supply system voltage & frequency.

(2) The transformers must be connected properly with regard to polarity. Polarity of the transformers must be Same.

(3) Voltage ratios of both primaries & Secondaries must be identical.

(4) The percentage Impedance should be equal in magnitude and have same X/R ratio (ratio of Reactance to Resistance) inorder to operate the transformers at different power factors.

(5) With transformers having different KVA ratings, the equivalent impedances should be proportional to the individual KVA rating.

Auto Transformer:

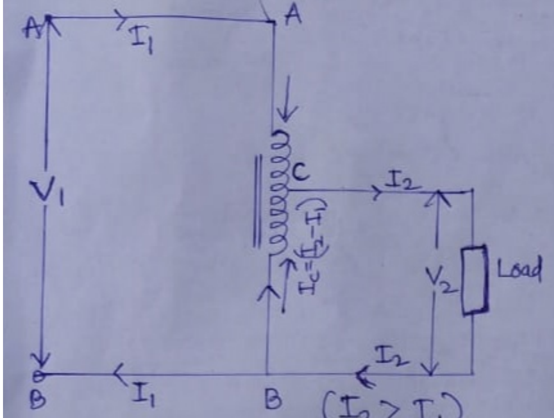
It consists of one winding only part of being common to both primary & Secondary. Power is transferred from primary to Secondary side by inductively & conductively. Two windings are not electrically isolated completely. Because of one winding its uses less copper & hence cheaper.

Uses – Auto transformers used when Transformation ratio 'K' is nearly equal to unity & there is no objection to electrical connection between primary & secondary.

1. as autotransformer Starters to start Induction Motor
2. as furnace transformer for supply to Induction furnace
3. ~~used~~ as booster to give small boost to distribution cable to correct voltage drop
4. as interconnecting transformers in 132/330 KV system

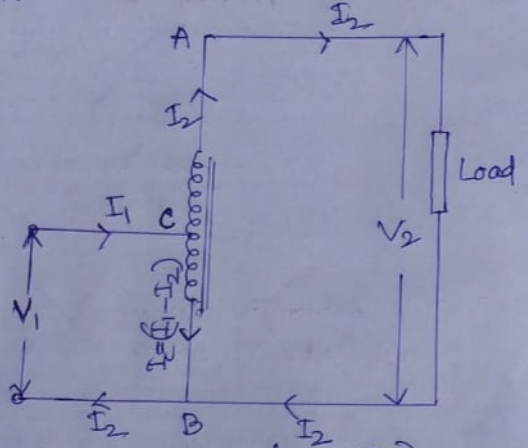
4. as interconnecting transformers in 132/330 KV Systems.
5. as control equipment for 1-phase & 3-phase electrical locomotives.

Working Principle of Auto Trf:-



$(I_2 > I_1)$

Step down Transformer



$(I_2 < I_1)$

Step up Transformer

for the Step down Transformer

AB = Primary winding having N_1 no of turns
BC = Secondary winding having N_2 no of turns
So Section of winding AC will have $(N_1 - N_2)$ no of turns.
V_1 = Input supply voltage, V_2 = Output Voltage across load Terminal
I_1 = Primary input current I_2 = output secondary current
Neglecting iron losses & no load current

Transformation ratio = $\dfrac{\text{Voltage across secondary winding (BC)}}{\text{Voltage across primary winding (AB)}}$

or $K = \dfrac{V_2}{V_1} = \dfrac{N_2}{N_1} = \dfrac{I_1}{I_2}$

Power Transferred Inductively ~~in case of step down Trf.~~

Input KVA $= V_1 I_1$
at secondary side power transferred
$= V_2 \times$ Current in Section BC $= V_2(I_2 - I_1)$
$= KV_1(I_2 - I_1) = KV_1\left(\dfrac{I_1}{K} - I_1\right)$
$= \dfrac{\cancel{K}V_1 I_1}{\cancel{K}} - \dfrac{KV_1 I_1}{} = V_1 I_1 - KV_1 I_1$
$= V_1 I_1 (1-K) = \text{Input}(1-K)$

Power Transfer Inductively in Case of Step up Trf.

Power transferred $= V_2$(current in Section BC) $= V_2(I_1 - I_2) = KV_1(I_1 - I_1/K)$
$= KV_1 I_1 - \dfrac{KV_1 I_1}{K} = V_1 I_1 (K-1)$
$= -V_1 I_1 (1-K) = \text{Input}(1-K)$
(-) sign as current flowing in Section BC is opposite to the figure for step down trf.

N.B – Section BC of the winding is only responsible for inductively power transfer as it is common to both primary & secondary side. Section AC is responsible for power transfer conductively.

The rest of power is directly transferred conductively from source to load.

i.e $V_1 I_1 - V_1 I_1 (1-K) = V_1 I_1 - V_1 I_1 + V_1 I_1 K = K V_1 I_1$

$= \text{Input} \times K$

Saving of Copper

Volume of copper winding is directly proportional to the length and area of cross section of the conductors.

Length $\propto$ No of turns (N)

Area of cross section $\propto$ magnitude of current (I)

So Volume $\propto NI$

& Weight $\propto NI$

Let W_o = Weight of copper in ordinary or two winding transformer

W_a = Weight of copper in auto transformer

Referring to the figure of step-down Auto Trf. we have

Weight of copper in section AC $\propto (N_1 - N_2) I_1$

" " " BC $\propto N_2 (I_2 - I_1)$

∴ total weight of copper in Auto Trf. $\propto$ section (AC + BC)

or $W_a \propto (N_1 - N_2) I_1 + N_2 (I_2 - I_1)$

or $W_a \propto (N_1 I_1 - N_2 I_1 + N_2 I_2 - N_2 I_1)$

or $W_a \propto (N_1 I_1 + N_2 I_2 - 2 N_2 I_1)$ ——— (1)

If a two winding transformer were to perform the same duty then

Weight of copper on its primary $\propto N_1 I_1$

Weight of copper on its secondary $\propto N_2 I_2$

Total weight of copper $W_o \propto (N_1 I_1 + N_2 I_2)$ ——— (2)

$\therefore$ $\dfrac{\text{Weight of Copper in Auto Transformer}}{\text{Weight of Copper in Ordinary Transformer}}$

$$= \frac{N_1 I_1 + N_2 I_2 - 2 N_2 I_1}{N_1 I_1 + N_2 I_2}$$

or $$\frac{W_a}{W_o} = 1 - \frac{2 N_2 I_1}{N_1 I_1 + N_2 I_2}$$

Dividing both Numerator & denominator by $N_1 I_1$ we have

$$\frac{W_a}{W_o} = 1 - \frac{\frac{2 N_2 \cancel{I_1}}{N_1 \cancel{I_1}}}{\frac{\cancel{N_1 I_1}}{\cancel{N_1 I_1}} + \frac{N_2 I_2}{N_1 I_1}}$$

or $$\frac{W_a}{W_o} = 1 - \frac{2 N_2/N_1}{1 + \left(\frac{N_2}{N_1}\right)\left(\frac{I_2}{I_1}\right)}$$

$$= 1 - \frac{2K}{1 + \cancel{K} \cdot \frac{1}{\cancel{K}}}$$ As $N_2/N_1 = \frac{I_1}{I_2} = K$

$$= 1 - \frac{2K}{2} = (1 - K)$$

or $$\boxed{W_a = W_o (1 - K)}$$

Saving in Copper $= W_o - W_a = W_o - W_o(1-K)$

$= \cancel{W_o} - \cancel{W_o} + W_o K = W_o K$

$\therefore$ Saving $= K \times$ Weight of Copper in Ordinary Transformer