DYNAMICS

* **Statics** → It is the branch of science which deals with forces and their effects when body is at rest.

* **Dynamics**: → It's the branch of science which deals with forces and their effects when body is at motion.

It is of two types:-

(a) Kinetics (b) Kinematics.

* **Kinetics**: → It is the branch of dynamics which deals with bodies in motion due to application of force.

* **Kinematics**: → It is the branch of dynamics which deals with bodies in motion without considering the effect of forces.

Principle of Dynamics:—

Dynamics concerned with the relationship between the motion of the particle and the force producing it. It gives us the second law of motion.

Before understanding Newton's law of motion let's understand some terms associated with linear motion.

1.) Speed → The speed of the body may be defined as rate of change of distance with respect to its surrounding.

$$\boxed{\text{Speed } (s) = \frac{\text{distance } (x)}{\text{time. } (t)}}$$

2.) Velocity → The velocity of a body may be defined as rate of change of displacement with respect to surrounding.

Displacement ⟶ Shortest distance.
↓
also denoted by (x)

$V = \frac{dx}{dt}$ ⟶ Change in displacement
⟶ change in time.

3) Acceleration :⟶ It's defined as the rate of change in its velocity.

$$\boxed{a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}}$$

* Uniform acceleration →

If velocity changes in equal magnitude in equal interval of time.

Equation of motion: →

1. $v = u + at$
2. $s = ut + \frac{1}{2}at^2$
3. $v^2 - u^2 = 2as$

v = final velocity in m/s

u = original velocity.

a = acceleration in m/s^2

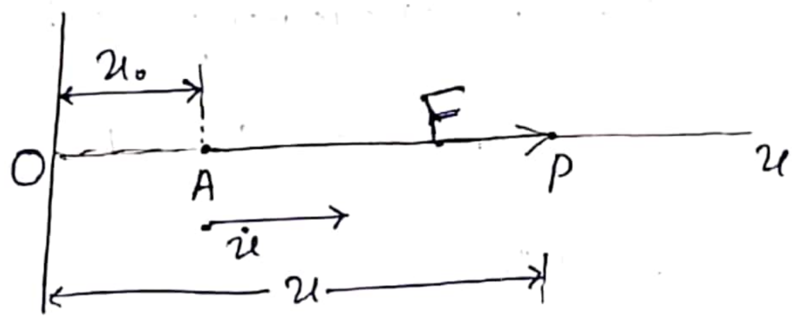
s = displacement in m

t = time in sec

* MOTION OF A PARTICLE ACTED UPON BY A CONSTANT FORCE :

→ This is second kind of dynamics problem. in which the acting force is given and resulting motion of the particle is required.

→ A particle acted upon by a constant force, the direction of which remain unchanged. In this case the particle moves rectilinerly in the direction of of force with a constant acceleration.



Let the line of motion belong x-axis. The magnitude of force be "F".

If particle start from rest means $t=0$ at position x_0 from origine.

Equation of motion,

$$\boxed{F = \frac{W}{g} a = \frac{W}{g} \ddot{x}}$$

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or, $\boxed{\ddot{x} = \frac{x}{W/g} = \frac{d^2x}{dx^2}}$ — (i)

To find the velocity V (or, $\dot{x}$) and displacement x as function af time

$\because \quad V = \frac{dx}{dt}$

$a = \frac{dv}{dt}$

$\therefore \quad \frac{d^2x}{dt^2} = a$

$\frac{dx}{dt} = at + C_1$

$\boxed{x = S = C_1 t + \frac{1}{2}at^2 + C_2}$ [S → space or displacement

at $t=0$, initial velocity $= u$ (assume)

$\therefore \quad \left.\frac{dx}{dt}\right|_{t=0} = u$

$\therefore \quad C_1 = u$

at $t=0$ let $x=0$; $\boxed{C_2 = 0}$

$\therefore \quad \boxed{S = ut + \frac{1}{2}at^2}$ — (A)

$\rightarrow \because \frac{dv}{dt} = a$

$v = at + c_1$

at $t=0$, $v = u$ (initial velocity)

$\therefore \boxed{v = u + at}$ — (B)

*** $\because$ from (B). $\boxed{at = (v-u)}$ — (i)

& Average velocity $= \frac{\text{total displacement}}{\text{total time}}$.

$\Rightarrow \left(\frac{v+u}{2}\right) = \frac{s}{t}$

$\Rightarrow \boxed{v+u = \frac{2s}{t}}$ — (ii)

from (i) × (ii)

$(v+u)(v-u) = 2as$

$\Rightarrow \boxed{v^2 - u^2 = 2as}$ — (C)

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Newton's law of mation:→

ⓐ Newton's first law af mation:→

Every body continues in its state af rest or af uniform mation in a straight line unless it is acted upon by some external force.

It is also called law af Inertia.

1→ A body at rest continues to remain at rest unless external force acts on it

Eg.→ A book lying on a table remain at rest it is lifted or pushed.

2.→ A body is moving with uniform velocity continues its state af uniform mation in a straight line unless an external force changes its mation

A body at rest has tendency to remain at Rest, it is called inertia af rest.

A body is in uniform mation in a straightline. has a tendency to presene its mation is called Inertia af mation.

(b) Newton's second law af mation:→

The rate af change af momentum is directly proportional to the impressed force and acts in the same direction af applied force.

9f.

m = mass of the body.

u = initial velocity of the body.

v = final velocity of the body.

a = Constant acceleration.

t = time in sec required to Change in velocity.

F = Force required to change velocity (from u to v)

Initial momentum $= mu$; final momentum $= mV$

$\boxed{\text{Change in momentum} = mv - mu.}$

Rate of change of momentum $= \dfrac{\text{change in momentum}}{\text{time interval.}}$

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma.$$

$\therefore$ A/c to Newton's second law of motion,

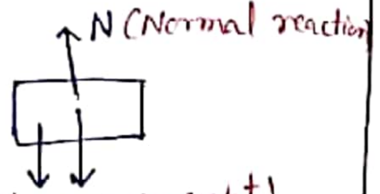
$F \propto ma$; $\underbrace{F = kma}_{k:\ \text{unit Constant of proportionality}}$; $\Rightarrow$ $\boxed{F = ma}$

$\boxed{\text{Unit of force} :\rightarrow \text{Newton}}$

* MOTION OF LIFT

(a) LIFT UPWARD

FBD:-



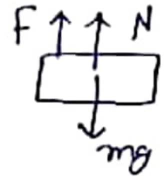
EQUATION:-

$F + mg = N$

$ma + mg = N$

$\boxed{m(a+g) = N}$

(b) LIFT DOWNWARD



(INERTIA "F") will be upward,

$F + N = mg$

$F - mg = -N$

$\therefore \boxed{N = m(g-a)}$

$\because F = ma.$

D'ALEMBERT'S PRINCIPLE :-

It states " if a rigid-body is acted upon by a system of forces is in dynamic equillibrium then. the sum of applied forces and inertia force is zero.

$$\because F = ma$$

$$F - ma = 0$$

* Newton's Third law of motion:-

It states that to every action there is always an equal and oppasite reaction.

Action means applied force which one body exerts on anather and reaction means body 2 exerts same (Body 1 $\rightarrow$ Body 2) amount of force on body 1.

Exampl... *A body has 50 kg mass on the earth. Find its weight (a) on the earth, where $g = 9.8 \ m/s^2$; (b) on the moon, where $g = 1.7 \ m/s^2$ and (c) on the sun, where $g = 270 \ m/s^2$.*

Solution. Given: Mass of body (m) = 50 kg ; Acceleration due to gravity on earth (g_e) = 9.8 m/s^2;Acceleration due to gravity on moon (g_m) = 1.7 m/s^2 and acceleration due to gravity on sun (g_s) = 270 m/s^2.

(*a*) *Weight of the body on the earth*

We know that weight of the body on the earth

$$F_1 = mg_e = 50 \times 9.8 = 490 \text{ N} \quad \textbf{Ans.}$$

(*b*) *Weight of the body on the moon*

We know that weight of the body on the moon,

$$F_2 = mg_m = 50 \times 1.7 = 85 \text{ N} \quad \textbf{Ans.}$$

(*c*) *Weight of the body on the sun*

We also know that weight of the body on the sun,

$$F_3 = mg_s = 50 \times 270 = 13500 \text{ N} = 13.5 \text{ kN} \quad \textbf{Ans.}$$

Example *A body of mass 7.5 kg is moving with a velcoity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.*

Solution. Given: Mass of body = 7.5 kg ; Velocity (u) = 1.2 m/s ; Force (F) = 15 N and time (t) = 2 s.

We know that acceleration of the body

$$a = \frac{F}{m} = \frac{15}{7.5} = 2 \text{ m/s}^2$$

$\therefore$ Velocity of the body after 2 seconds

$$v = u + at = 1.2 + (2 \times 2) = 5.2 \text{ m/s} \quad \textbf{Ans.}$$

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Solution. Given: Retarding force (F) = 50 N ; Mass of the body (m) = 20 kg ; Initial velocity (u) = 15 m/s and final velocity (v) = 0 (because it stops)

Let t = Time taken by the body to stop.

We know that retardation of the body

$$a = \frac{F}{m} = \frac{50}{20} = 2.5 \text{ m/s}^2$$

and final velocity of the body,

$$0 = u + at = 15 - 2.5\,t \qquad \text{...(Minus sign due to retardation)}$$

$$\therefore \qquad t = \frac{15}{2.5} = 6 \text{ s} \qquad \textbf{Ans.}$$

Example *A car of mass 2.5 tonnes moves on a level road under the action of 1 kN propelling force. Find the time taken by the car to increase its velocity from 36 km. p.h. to 54 km.p.h.*

Solution. Given : Mass of the car (m) = 2.5 t ; Propelling force (F) = 1 kN ; Initial velocity (u) = 36 km.p.h. = 10 m/s and final velocity (v) = 54 km.p.h. = 15 m/s

Let t = Time taken by the car to increase its speed.

We know that acceleration of the car,

$$a = \frac{F}{m} = \frac{1}{2.5} = 0.4 \text{ m/s}^2$$

and final velocity of the car (v),

$$15 = u + at = 10 + 0.4\,t$$

$$t = \frac{15 - 10}{0.4} = \frac{5}{0.4} = 12.5 \text{ s} \qquad \textbf{Ans.}$$

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Example *A body of mass 50 kg is being lifted by a lift in an office. Find the force exerted by the body on the lift floor, when it is moving with a uniform acceleration of 1.2 m/s².*

Solution. Given : Mass of the body (m) = 50 kg and acceleration (a) = 1.2 m/s²

We know that pressure exerted by the body on the floor, when it is being lifted

$$F = m\,(g + a) = 50\,(9.8 + 1.2) = 550 \text{ N} \quad \textbf{Ans.}$$

Example *In a factory, an elevator is required to carry a body of mass 100 kg. What will be the force exerted by the body on the floor of the lift, when (a) the lift is moving upwards with retardation of 0.8 m/s²; (b) moving downwards with a retardation of 0.8 m/s².*

Solution. Given : Mass of the body (m) = 100 kg and acceleration (a) = – 0.8 m/s² (Minus sign due to retardation)

(a) *When the lift is moving upwards*

We know that force exerted by the body on the floor of the lift

$$F_1 = m\,(g + a) = 100\,(9.8 - 0.8) = 900 \text{ N} \quad \textbf{Ans.}$$

(b) *When the lift is moving downwards*

We also know that force exerted by the body on the floor of the lift.

$$F_2 = m\,(g - a) = 100\,(9.8 + 0.8) = 1060 \text{ N} \quad \textbf{Ans.}$$

Exampl *An elevator is required to lift a body of mass 65 kg. Find the acceleration of the elevator, which could cause a force of 800 N on the floor.*

Solution. Given : Mass of the body (m) = 65 kg and Force (R) = 800 N

Let a = Acceleration of the elevator.

We know that the force caused on the floor when the elevator is going up (R),

$$800 = m\,(g + a) = 65\,(9.8 + a)$$

or

$$a = \frac{800}{65} - 9.8 = 2.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

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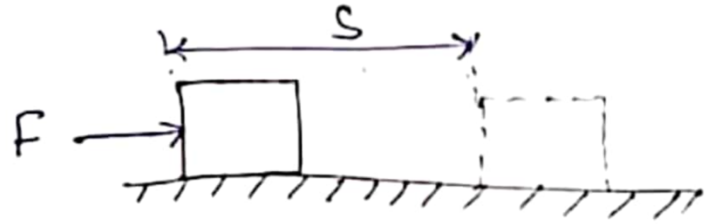
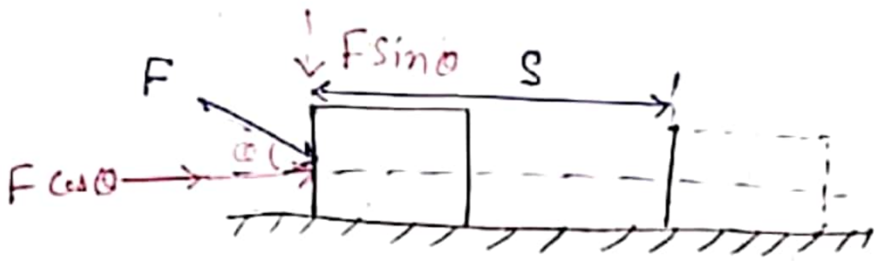
***** WORK, ENERGY AND POWER

Work $\longrightarrow$ When a force acting on a body and the body undergoes a displacement then some work is said to be done. Thus the work done by a force on a moving body is defined as the product of the force and distance moved in the direction of the force. SI unit:- N-m = J



Body moving in the direction of force

$\therefore$ work, $\boxed{W = F \times S}$



Body is not moving in the direction of force so, we consider $F\cos\theta$ (component of F) in the direction of force.

$\therefore$ work, $\boxed{W = F\cos\theta \times S}$

Energy: → Energy is defined as the Capacity of doing work. There are many forms of energy like Heat energy, mechanical energy, electrical energy etc. In mechanics mast useful energy is mechanical energy.

Mechanical Energy may be classified into two Forms :-

(i) patential Energy (ii) Kinetic Energy.

- Patential Energy → The Capacity to do work due to pasition af the body. A body is said Stable equillibrium if ther net force is zero and small changes in the System would Cause increase in patential energy.

  A body with weight W at height h will ~~pases~~ passesses an energy $\boxed{U = W \times h}$

- Kinetic Energy → Capacity af doing work due to mation af the body is called Kinetic energy.

$$K.E = \frac{1}{2} \times m \times v^2$$

m = mass af the body

v = velocity af the body.

or, $\boxed{K.E = \frac{1}{2} m v^2}$

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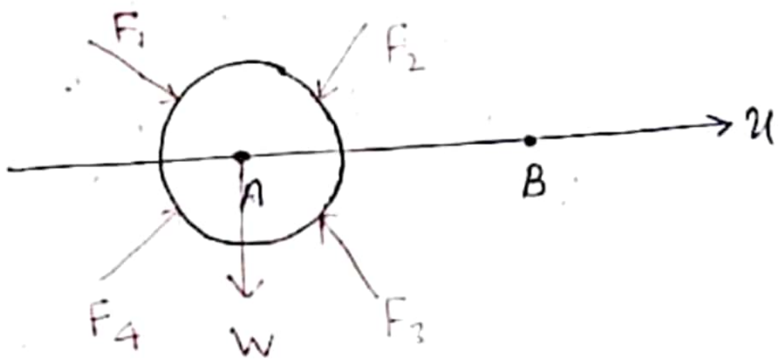
POWER ⟶ Time rate of doing work is called power. Unit of power is watt (W), i.e. one jule of work in one second.

Work Energy equation :⟶



Consider a body "A". Forces acting as shown in figure.

From Newton's second law,

$$\boxed{F = \frac{W}{g} a}$$ [a be acceleration]

multiply element 'ds' bath sides.

$$\therefore \quad F\,ds = \frac{W}{g}\frac{dv}{dt} \times ds \qquad \left[\because \frac{dV}{dt} = a\right]$$

$$\Rightarrow \quad F\,ds = \frac{W}{g}\,dv \times \frac{ds}{dt} = \frac{W}{g} \times V\,dv$$

$$\left[\because \frac{ds}{dt} = V\right]$$

Integrating bath sides

$$\int_0^s F\,ds = \int_u^V \frac{W}{g} \times V\,dv$$

$$\therefore \quad F.S = \frac{W}{2g}\left[V^2 - u^2\right] = \text{Final K.E} - \text{Initial KE}$$

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D'ALEMBERT'S PRINCIPAL

It states " If a body is acted upon by a system of forces, this system may be reduced to a single resultant Force whose magnitude, direction and the line of action may be found out by the method of graphic statics".

$$\boxed{P = ma} \quad \text{— (i)}$$

[P is vector sum of forces and a is acceleration of the body]

This equation may be written as

$$\boxed{P - ma = 0} \quad \text{— (A)}$$

→ Equation (i) is the equation of dynamics while eq. (A) is 'equation of statics'

→ Equation (A) is known as the equation of dynamic equillibrium under the action of real force P.

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LAWS OF MOTION QUESTIONS (LOM.1)

Qn:- A body has 50kg mass on the earth. Find its weight (a) on the earth where $g = 9.81 m/s^2$ (b) on the moon 1.7 (c) on the sun, $g_s = 270 m/s^2$.

Soln.→ Given, mass of the body, $m = 50 kg$,

acceleration due to gravity on earth, $g_e = 9.8 m/s^2$

" " " " " moon, $g_m = 1.7 m/s^2$

" " " " " Sun, $g_s = 270 m/s^2$

(a) weight on the earth;

$F_1 = mg_e = 50 \times 9.8 = 490 N$ ANS.

(b) Weight on the moon,

$F_2 = mg_m = 50 \times 1.7 = 85 N$ ANS.

(c) weight on the Sun,

$F_3 = mg_s = 50 \times 270 = 13500 N$ ANS.

Qn- A body of mass 7.5 kg moving with velocity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.

Soln:→ Given, $m = 7.5 kg$; $u = 1.2 m/s$, $F = 15 N$

& $t = 2 s$.

we know, $a = \frac{F}{m} = \frac{15}{7.5} = 2 m/s^2$

and velocity of the body after 2 s.

$\boxed{v = u + at = 1.2 + (2 \times 2) = 5.2 m/s}$ — Ans

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Qn:- A body of mass 50kg is being lifted by a lift in an office. Find the force exerted by the body on the lift floor, when it is moving with a uniform acceleration of 1.2m/s

Soln:→ Given, mass, $m = 50kg$,

acceleration; $a = 1.2m/s$

we know, $\boxed{F = m(g+a) = 50 \times (9.8+1.2) = 550 N}$

Qn:- In a factory, an elevator is required to carry a body of mass 100kg. What will be the force exerted by the body on the floor of the lift. If

(a) The lift is moving upword with retardation of $0.8 m/s^2$

(b) moving downward with retardation of $0.8 m/s^2$

Soln:- (a) for upward, $F_1 = m(g+a) = 100 \times (9.8 - 0.8)$

$\therefore \boxed{F_1 = 100 \times 9 = 99 N}$ — ANS.

(b) for downward, $F_2 = m(g-a) = 100 \times (9.8 - (-0.8))$

$\boxed{F_2 = 100 \times 10.6 = 1060 N}$ — ANS.

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Q) A stone is thrown upwards with a velocity 4.9 m/s from a bridge. If it falls down in water after 2 s. then find the height of the bridge.

Solution:- Given,

(negative upward) initial velocity, $u = -4.9 m/s$ (- singn for UPWARD)

and the time, $t = 2 s$.

$$\therefore \quad h = ut + \frac{1}{2}gt^2 \quad (a = g \text{ for free fall})$$

$$= (-4.9 \times 2) + \frac{1}{2} \times 9.81 \times (2)^2$$

$$\boxed{h = (-9.8 + 19.6) m = 9.8 m \text{ Ans.}}$$

Qn:- A packet is dropped from a balloon which is going upwards with a velocity 12 m/s. Calculate the velocity of packet after 2 s. (negative upward)

Soln:- Given, $u = -12 m/s$ ('−' for UPWARD)

$t = 2 s$.

$\therefore$ velocity of packet after 2 s,

$$\boxed{V = u + gt = -12 + 9.81 \times 2 = 7.6 m/s.}$$

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Qn:- A stone is dropped from the top of a building which is 65m height. With what velocity will it hit the ground?

Soln:→

Initial velocity, $u=0$ (Started from rest)

and height of the ground, $S = 65m$.

we know that, $v^2 = u^2 + 2gs$

$= (0)^2 + 2 \times 9.81 \times 65 = 1274$

$$\boxed{\therefore V = \sqrt{1274} = 35.7 m/s}$$

Qn:- A body is thrown vertically upward with a velocity of 28 m/s. Find the distance it will cover in 2 sec.

Soln:-

$u = -28$ m/s (negative means upward vel.)

$t = 2s$

We know that

$S = ut + \frac{1}{2}gt^2$

$= (-28 \times 2) + \frac{1}{2} \times 9.81 \times (2)^2$

$\boxed{S = -36.4m.}$ → (negative means upward)

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Question on Work Power & Energy

Question:– A Truck of mass 15 tonnes travelling 1.6 m/s impacts with buffer spring which Compresses 1.25 mm per KN. Find the maximum Compression of the spring.

Sol^n: → Given,

$m = 15\,t$, $V = 1.6\,m/s$, $L_K = 1.25\,mm/KN$

$\left(L_K = \frac{1}{\text{Spring const}} \right)$

To find:–

x = Max^m Compression of the spring.

we know that Kinetic energy,

$$U = \frac{1}{2} m v^2 = \frac{1}{2} \times 15 \times (1.6)^2 = 19\,200\ KN\text{-}mm \quad \text{—(I)}$$

and Compressive load, $= \frac{x}{1.25} = 0.8x\ KN$

work done in Compressing the spring.

$=$ Average Compressive load $\times x$

$$= \frac{0.8x}{2} \times x = 0.4x^2\ KN\text{-}mm. \quad \text{—(II)}$$

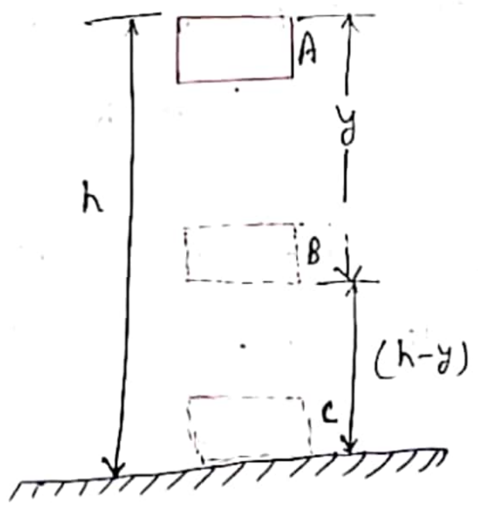
∵ Kinetic energy = Compression work of spring.

$$19200 = 0.4x^2$$

$\boxed{x = 219\,mm}$ – ANS.

* TRANS-FORMATION OF ENERGY

→ we studied about kinetic and patential energy. here we are gaing to discuss about transformatio of patential energy into kinetic energy.



In figure consider a body just dropped on the ground from pasition A to ground C.

Let mass of the body = m

height from the body is dropped = h

AT-A

∵ No initial velocity at "A"

∴ Kinetic energy at A = 0 (K.E = 0)

and patential energy at A = mgh. (PE = mgh)

Tatal energy at A = mgh. (TE)

AT-B

B is at a distance y from top so

velocity at B = $\sqrt{2gy}$

Kinetic energy at B = $\frac{1}{2}mv^2 = \frac{1}{2} \times m \times (\sqrt{2gy})^2$

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$\therefore$ Kinetic energy of the body at $B = mgy$

and patential energy at $B = mg(h-y)$ [$\because (h-y)$ distance from the ground]

$\therefore$ Tatal Energy at $B = mgy + mg(h-y)$

$= mgh$

AT-C

We know that at C the body has fallen through a distance (h). Therefore velocity of the body at $C = \sqrt{2gh}$

$\therefore$ Kinetic energy at $C = \frac{1}{2} \times m \times (\sqrt{2gh})^2$

$= mgh$

and patential energy at $C = 0$ ($\because h=0$ from ground

$\therefore$ Tatal Energy at $C = mgh$.

Conclusion:→ Tatal energy (sum of Kinetic and patenti[al] remans Constant at all paints. but transformati[on] of K.E & potential energy is obvious passible.

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LAW OF CONERVATION OF ENERGY :—

It states " The energy can neither be created nor destroyed, though it can be transformed from one form into any of the form in which energy can exist."

As in previous problem we proved the interchanging of K.E & P.E.

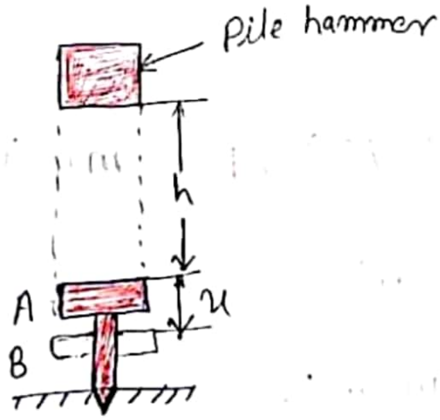
The above statement may be exemplified as below:-

(1.) Electrical heater ⟶ electrical energy to heat energy

(2.) Electric bulb ⟶ electrical energy to light energy.

(3.) In dynamo ⟶ Mechanical energy to electrical energy.

* PILE AND PILE HAMMER



A pile and pile hammer is shown in the above figure. The pile hammer has potential energy before released. After release, the potential energy is converted into kinetic energy, with which the hammer will strike the pile.

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Let m = mass of the pile hammer
M = mass of the pile.
h = height through which the pile hammer falls before striking the pile.
x = distance through which the pile deeps into the ground.
u = velocity of the pile hammer before strike.
V = velocity (common) of the pile and pile hammer after impact. ($= 0$ at distance x)
R = average resistance of the soil.

$\therefore$ $\boxed{u = \sqrt{2gh}}$ — before striking.

momentum of the pile hammer just before strike,

$$\boxed{P = mu} \text{ — (i)}$$

and just after impact, $\boxed{P = (M+m)V}$ — (ii)

from (i) & (ii)

$$mu = (M+m)V$$

$$\therefore \boxed{V = \frac{mu}{M+m}} \text{ — (iii)}$$

energy of pile and pile hammer just after impact

$$= KE = \frac{1}{2}(m+M)V^2$$

& $P.E = (m+M)x \times g$ ($x \to$ distance from datum)

Total energy, $T.E = K.E. + P.E.$

$$= \frac{(m+M)v^2}{2} + (m+M)gx$$

Substituting the value of V from (iii)

$$T.E = \frac{m^2u^2}{2(M+m)} + (m+M)gx$$

Now, $u^2 = (\sqrt{2gh})^2 = 2gh.$

$$\therefore \quad T.E = \frac{m^2 \times 2gh}{2(M+m)} + (M+m)gx$$

or, $$\boxed{T.E = \frac{m^2gh}{(M+m)} + (M+m)gx}$$

If soil resistance = R.

then energy due to soil resistance $= Rx = T.E.$

$$Rx = \frac{m^2gh}{(M+m)} + (M+m)g \times x$$

$$\therefore \quad R = \frac{m^2gh}{x(M+m)} + (m+M)g.$$

Sometimes pile mass M is neglected.

$$\therefore \quad \boxed{R = \frac{m^2gh}{xm} + mg = mg\left(\frac{h}{x}+1\right)}$$

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Qn:→ A pile of negligible mass is ~~der~~ driven by a hammer of mass 200kg. If the pile is driven 500mm into the ground, when the hammer falls from a height of 4m. Find the average force of resistance of the ground. (m is not given so neglect)

Soln:- Given, $m = 200kg$, $x = 500mm$, $h = 4m$.

$$\therefore \boxed{R = mg\left(\frac{h}{x}+1\right) = 200 \times 9.8 \times \left(\frac{4}{0.2}+1\right) = 17640N}$$

ANS.

Qn:→ A hammer of mass 0.5 kg hits the nail of 25g with a velocity of 5m/s and drives into a fixed wooden block by 25mm. Find the resistance offered by the ground.

Soln:- Given, $m = 0.5kg$, $M = 25g = 0.025kg$, $u = 5m/s$

$x = 0.025m$. $h \rightarrow$ unknown.

$$\therefore 5 = \sqrt{2gh}$$

$$\Rightarrow h = \frac{5^2}{2 \times 9.8} = 1.28m.$$

$$\therefore R = \frac{m^2 gh}{x(m+M)} + (m+M) \times g$$

$$= \frac{(0.5)^2 \times 9.8 \times 1.28}{0.025 \times (0.5+0.025)} + (0.5+0.025) \times 9.8$$

$\boxed{R = 244N}$ — ANS.

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*** COLLISION OF ELASTIC BODIES (COEB)

→ The property of bodies, by virtue of which The rebound, after impact is called elasticity.

→ The height of the body after rebound is more means more elastic less means more plastic.

* PHENOMENON OF COLLISION :-

When two elastic bodies collide with each other, the phenomenon of collision takes place as given below:-

1) The bodies, immediately after collision . h comes momentarily to rest

2.) The Two bodies tend to compress each other, so long as they are compressed to the maximum value.

3.) The two bodies attempt to regain its original shape due to their elasticity. This process of Regaining the original shape is called restitution.

* LAWS OF CONSERVATION OF MOMENTUM.

It states " The total momentum of two bodies remains constant after their collision or any other mutual action".

i.e.

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$$

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m_1 = mass of initial body; m_2 = mass of final body

V_1, V_2 = final velocities of first and second body respectfully.

u_1, u_2 = Initial velocities of First and second body.

Note:— (i) momentum is product of mass and velocity.

(ii) $m_1 v_1 \rightarrow$ First body momentum after callision.

$m_1 u_1 \rightarrow$ First body momentum before callision

So as second body $m_2 V_2$ & $m_2 u_2$.

NEWTON'S LAW OF COLLISION FOR ELASTIC BODIES

It states," when two moving bodies callide with each ather, their velocity af separation bears a constant ratio to their velocity af approach."

ie.

$$\boxed{(V_2 - V_1) = e\,(u_1 - u_2)} \quad \text{— (A)}$$

$V_2, V_1 \longrightarrow$ Final velocities af body 2 & body 1.

$u_2, u_1 \longrightarrow$ Initial velocities af body 2 & body 1.

$e \rightarrow$ Constant af proportionality.

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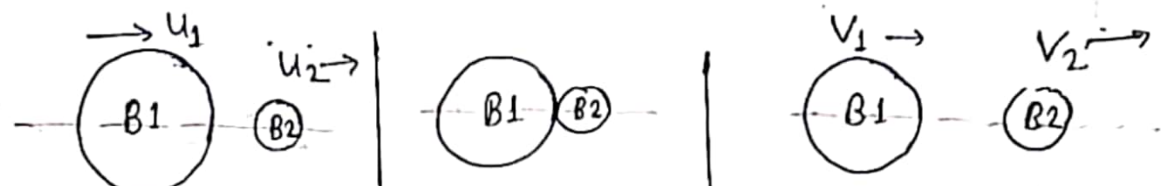
COEB-3

COEFFICIENT OF RESTITUTION (e) (B_1 & B_2 → Body 1 & Body 2)



a) Before impact

ⓑ During Callision

ⓒ After collision.

approach velocity
$= (u_1 - u_2)$

Separation velocity
$= (V_2 - V_1)$

Velocity af app-raach → relative velocity befere Callision
$= (u_1 - u_2)$

velocity af separation $= (V_2 - V_1)$

From Ⓐ

coefficient af restitution, $e = \dfrac{\text{velocity af Separation}}{\text{velocity af approach}}$

$$\therefore e = \left(\frac{V_2 - V_1}{u_1 - u_2}\right)$$

Note → (1) range af e, $\boxed{0 \le e \le 1}$

(2) when $e = 0$ the two bodies are <u>inelastic</u>

(3) when $e = 1$ the two bodies are perfectly plastic

(4) ALL VELOCITIES HAVE SAME DIRECTIONS FOR

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COEB-4.

FOR OPPOSITE DIRECTION CONSIDER ONE OF THEM NEGATIVE.

(4) Above formula is used for $u_1 > u_2$ & $v_2 > v_1$ but for special case like $u_2 > u_1$ or $v_1 > v_2$ in the equation e will be negative so take modulus and make e positive value.

* TYPES OF COLLISION → TWO TYPES

(1) Direct impact, and (2) Indirect (or, oblique) Impact.

→ our scope is to study direct impact only.

→ In above explanation we discussed about conservation of momentum which is only for direct impact.

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Example . .. *A ball of mass 1 kg moving with a velocity of 2 m/s impinges directly on a* *ıll of mass 2 kg at rest. The first ball, after impinging, comes to rest. Find the velocity of the second* *ıll after the impact and the coefficient of restitution.*

Solution. Given : Mass of first ball (m_1) = 1 kg ; Initial velocity of first ball (u_1) = 2 m/s ; lass of second ball (m_2) = 2 kg ; Initial velocity of second ball (u_2) = 0 (because it is at rest) and final elocity of first ball after impact (v_1) = 0 (because, it comes to rest)

elocity of the second ball after impact.

Let v_2 = Velocity of the second ball after impact.

We know from the law of conservation of momentum that

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(1 \times 2) + (2 \times 0) = (1 \times 0) + (2 \times v_2)$$

$$\therefore \quad 2 = 2v_2$$

or $\quad v_2 = 1$ m/s **Ans.**

oefficient of restitution

Let e = Coefficient of restitution.

We also know from the law of collision of elastic bodies that

$$(v_2 - v_1) = e\,(u_1 - u_2)$$

$$(1 - 0) = e\,(2 - 0)$$

or $\quad e = \dfrac{1}{2} = 0.5$ **Ans.**

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Example . *The masses of two balls are in the ratio of 2 : 1 and their velocities are in the ratio of 1 : 2, but in the opposite direction before impact. If the coefficient of restitution be 5/6, prove that after the impact, each ball will move back with 5/6th of its original velocity.*

Solution. Given : Mass of first ball (m_1) = 2 M ; Mass of second ball (M_2) = M ; Initial velocity of first ball (u_1) = U ; Initial velocity of second ball (u_2) = $-2U$ (Minus sign due to opposite direction) and coefficient of restitution (e) = $\frac{5}{6}$

Let v_1 = Final velocity of the first ball, and

v_2 = Final velocity of the second ball.

We know from the law of conservation of momentum that

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$2M \times U + M(-2U) = 2Mv_1 + Mv_2$$

or $$0 = 2Mv_1 + Mv_2$$

$\therefore$ $$v_2 = -2v_1 \qquad ...(i)$$

We also know from the law of collision of elastic bodies that

$$(v_2 - v_1) = e\,(u_1 - u_2) = \frac{5}{6}\left[U - (-2U)\right] = \frac{5U}{2} \qquad ...(ii)$$

Substituting the value of v_2 from equation (i)

$$\left[-2v_1 - (v_1)\right] = \frac{5U}{2} \qquad \text{or} \qquad v_1 = -\frac{5}{6} \times U$$

Minus sign indicates that the direction of v_1 is opposite to that of U. Thus the first ball will move back with $\frac{5}{6}$th of its original velocity. **Ans.**

Now substituting the value of v_1 in equation (i).

$$v_2 = -2\left(-\frac{5}{6} \times U\right) = +\frac{5}{6} \times 2U$$

Plus sign indicates that the direction of v_2 is the same as that of v_1 or opposite to that of u_2. Thus the second ball will also move back with $\frac{5}{6}$th of its original velocity. **Ans.**

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Example .... *Three perfectly elastic balls A, B and C of masses 2 kg, 4 kg and 8 kg move in the same direction with velocities of 4 m/s, 1m/s and 0.75 m/s respectively. If the ball A impinges with the ball B, which in turn, impinges with the ball C, prove that the balls A and B will be brought to rest by the impacts.*

Solution. Given : Coefficient of restitution (e) = 1(because the balls are perfectly elastic) : Mass of ball A (m_1) = 2 kg ; mass of ball B (m_2) = 4 kg ; Mass of ball C (m_3) = 8 kg ; Initial velocity of ball A (u_1) = 4 m/s ; Initial velocity of ball B (u_2) = 1 m/s and initial velocity of ball C (u_3) = 0.75 m/s

Final velocity of the first ball after impact

First of all, consider the impact of the first and second ball.

Let v_1 = Final velocity of the first ball after impact, and

v_2 = Final velocity of the second ball after impact.

We know from the law of conservation of momentum that

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(2 \times 4) + (4 \times 1) = 2 \times v_1 + 4 \times v_2$$

$$2v_1 + 4v_2 = 12 \quad \text{or} \quad v_1 + 2v_2 = 6 \qquad ...(i)$$

We also know from the law of collision of elastic bodies that

$$(v_2 - v_1) = e\,(u_1 - u_2) = 1\,(4 - 1) = 3 \qquad ...(ii)$$

Adding equations (*ii*) and (*i*),

$$3v_2 = 9 \quad \text{or} \quad v_2 = 3 \text{ m/s}$$

Substituting the value of v_2 in equation (*ii*),

$$3 - v_1 = 3 \quad \text{or} \quad v_1 = 0$$

Thus the first ball will be brought to rest by the impact of first and second ball. **Ans.**

Final velocity of the second ball

Now consider the impact of second and third ball. In this case $u_2 = v_2 = 3$ m/s

Let v_2 = Final velocity of the second ball, after the impact of second and third ball, and

v_3 = Final velocity of the third ball after impact.

We know from the law of conservation of momentum that

$$m_2u_2 + m_3u_3 = m_2v_2 + m_3v_3$$

$$(4 \times 3) + (8 \times 0.75) = 4 \times v_2 + 8 \times v_3$$

$$4v_2 + 8v_3 = 18 \quad \text{or} \quad 2v_2 + 4v_3 = 9 \qquad ...(iii)$$

We also know from the law of collision of elastic bodies that

$$(v_3 - v_2) = e\,(u_2 - u_3) = 1\,(3 - 0.75) = 2.25$$

Multiplying the above equation by 4,

$$\therefore \quad 4v_3 - 4v_2 = 9 \qquad ...(iv)$$

Subtracting equation (*iv*) from (*iii*),

$$6v_2 = 0 \quad \text{or} \quad v_2 = 0$$

Hence the second ball will also be brought to rest by the impact of second and third ball. **Ans.**

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