Unit:-2 ÷ Scalar And Vectors

Scalar Quantities:-

* The physical quantity having only magnitude are called Scalar quantitgies.

Ex:- Mass, length, volume, density, Energy, temperature, Electric Charge etc.

Vector Quantities:-

* The physical quantities having both magnitude as well as direction are called vector quantities.

Ex:- Displacement, velocity, acceleration, force, electric Intensity, Magnetic intensity, Magnetic movement etc.

Representation Of A vector:-

* A vector is written with an arrow head over its symbol like '$\vec{x}$'.

* Examples:- Acceleration ($\vec{a}$).
Velocity ($\vec{v}$).
Force ($\vec{F}$)

Types Of vector:-

1) Null Vector:-

The vector having '0' magnitude and an Arbitary ~~constant~~ direction is called a Null vector.

2) Equal vector:-

Two vectors are said to be equal if they possess the same magnitude and direction.

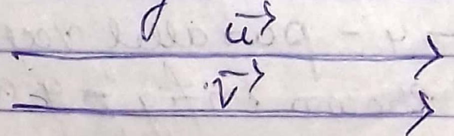


Fig:- 1:- Equal vectors

(iii) Negative Vector:-

A vector is said to be a negative vector of another one if it is represented by a line having same length as that of the second and is directed in opposite direction.

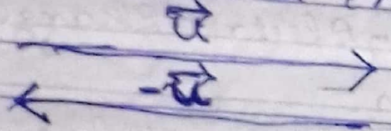


Fig:- 2:- Negative Vector.

(iv) Co- Initial Vector:-

A number of vectors having a common initial point are called co-initial vectors.

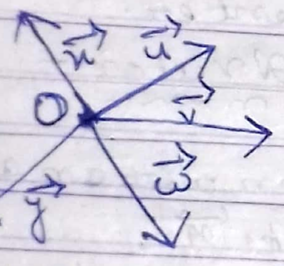


Fig:- 3:- Co - Initial vectors

(v) Collinear Vector:- Vectors having a common line of action are called collinear vector.

There are two types of collinear vectors.

a) parallel vector:- $(\theta = 0°)$

→ Two vectors (which may have different magnitudes) acting along same direction are parallel vectors.

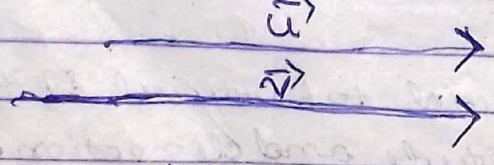


Fig:- 4:- parallel vectors

Vectors $\vec{u}$ and $\vec{v}$ shown in fig:- 4 are parallel and angle betn them is $0°$.

b) Anti parallel Vector :-

Two vectors which are directed in opposite directions are called Anti-parallel vectors.

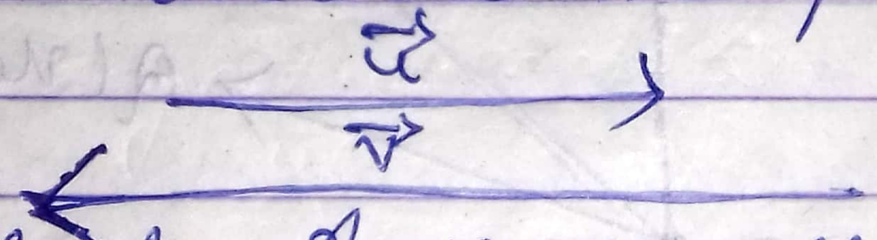


Fig :- 5 :- Anti-parallel vectors.

Vectors $\vec{u}$ and $\vec{v}$ shown in figure :- 5 are antiparallel vector and angle between them is $180°$.

(vi) Co-planar vector:-

vectors situated in one ~~place~~ plane irrespective of their directions are known as co-planar vectors.

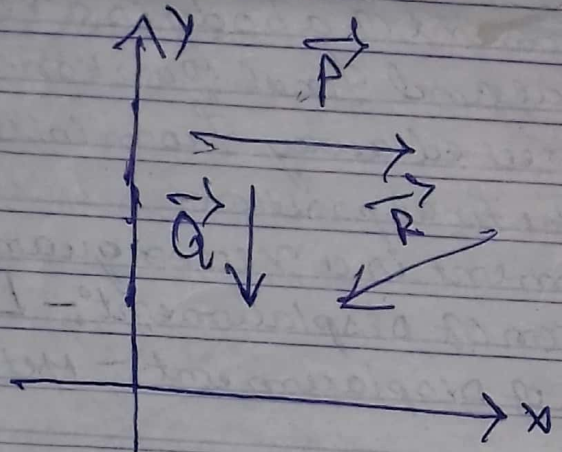


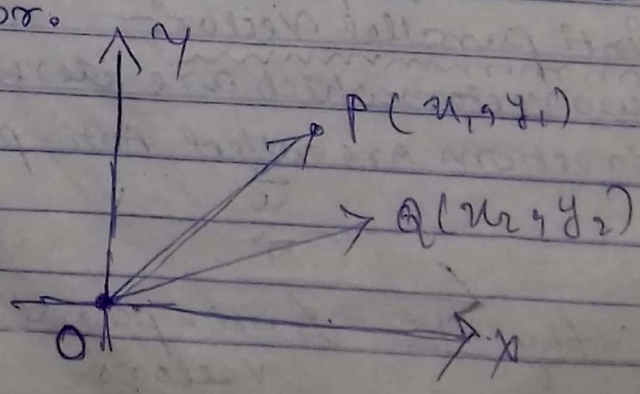
Fig:- 6:- Co-planar vector

Vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ drawn on xy plane are co-planar vectors.

(vii) Localised Vector:-

Vector whose initial point (tail) is fixed is said to be a localised or a fixed vector.

position vector of every point starts from origin, so it can be considered as a localised or fixed vector.



(viii) Non-Localised vector:-

Vector who's initial point (tail) is not fixed is said to be a non-localised vector or a free vector.
Vectors representing force, momentum etc. are Non-Localised vector.

Triangle Law of vector Addition:-

It is a law for the addition of two vectors. It can be stated as follows:- "If two vectors are represented (in magnitude and direction) by the two sides of a triangle taken in same order then their resultant is represented, (In magnitude and direction) by the third side of the triangle taken in opposite order."

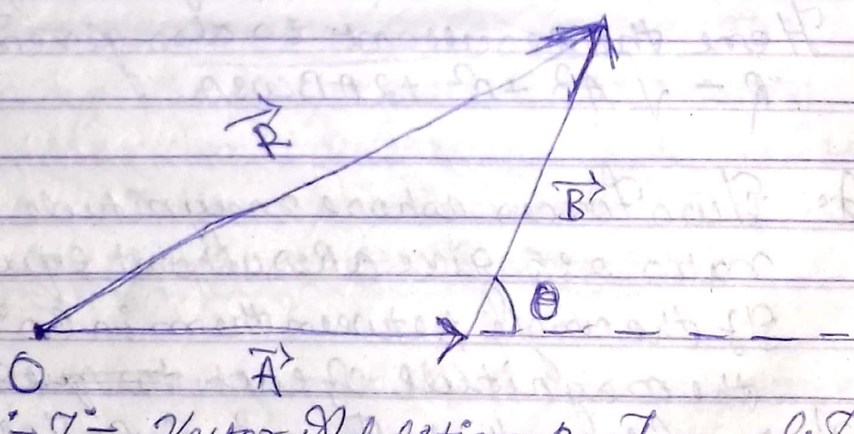


Fig:-7:- Vector Addition By Triangle Law

Resultant of two vectors $\vec{A}$ and $\vec{B}$ acting at a point can be determined by triangles law. The resultant is given by,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{(i)}$$

Where, 'θ' is the angle between $\vec{A}$ and $\vec{B}$.

Parallelogram Law of Vector Addition :-

"It states that if two vectors acting simultaneously at a point represented (in magnitude and direction) by the two sides of parallelogram drawn from a point then their resultant is given (in magnitude or direction) by the diagonal of the parallelogram passing through that point."

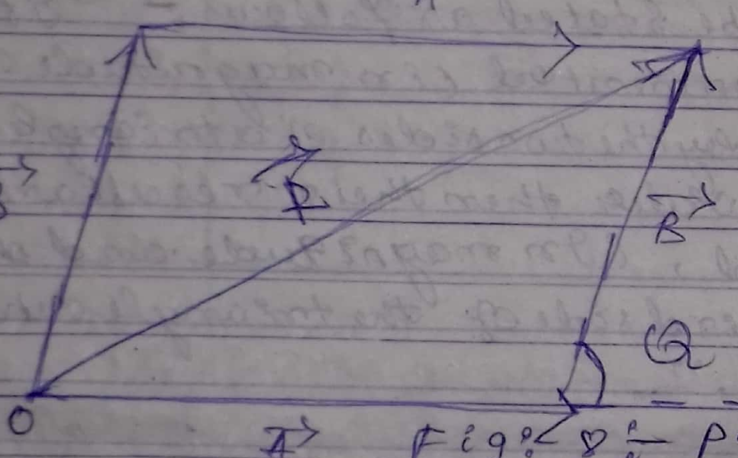


Fig:- 8 :- P. Law of Vector Addition

Here the Resultant is also given by

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Q. Two forces whose magnitude are in the ratio 3:5 give a Resultant equal to 70N. If the angle between them is 60°, find the magnitude of each ~~torque~~ force.

Ans:- Given that $\frac{F_1}{F_2} = \frac{3}{5}$; $R = 70N$; $\theta = 60°$

We know that $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$\Rightarrow R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$\Rightarrow (70)^2 = \left(\frac{3}{5}F_2\right)^2 + F_2^2 + 2\times\frac{3}{5}F_2 \times F_2 \times \cos 60°$

$\Rightarrow 4900 = \frac{9}{25}F_2^2 + F_2^2 + \frac{6}{5}F_2^2 \times \frac{1}{2}$

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$\Rightarrow 4900 = \frac{9}{25} F_2^2 + F_2^2 + \frac{3}{5} F_2^2$

$\Rightarrow 4900 = F_2^2 \left(\frac{9}{25} + \frac{3}{5} + 1\right)$

$\Rightarrow 4900 = F_2^2 \left(\frac{9}{25} + \frac{3}{5} + 1\right)$

$\Rightarrow 4900 = F_2^2 \left(\frac{9 + 15 + 25}{25}\right)$

$\Rightarrow 4900 = F_2^2 \left(\frac{49}{25}\right)$

$\Rightarrow F_2^2 = 4900 \times \frac{25}{49} = 2500$

$\Rightarrow F_2 = \sqrt{2500} = 50N$

$\therefore F_1 = \frac{3}{5} F_2 = \frac{3}{5} \times 50 = 30N$

Resolution of vectors in a plane:-

* Resolution of vectors is the process of obtaining the component of the vector which when combined according to the laws of vector addition produced the given vector.

Rectangular components of a vector:-

Rectangular components of a given vector are its components in two mutually per-pendicular directions in the plane of the given vector.

Let $\vec{OP}$ $(\vec{R})$ be the position vector of a point $P(x, y)$.

Applying $\triangle$ law in $\triangle OAP$.

$\vec{OP} = \vec{OA} + \vec{AP}$

$\Rightarrow \vec{R} = \vec{OA} + \vec{OB}$

$= x\hat{i} + y\hat{j}$

In ΔOAP,

$\therefore \cos\theta = \frac{b}{h} = \frac{OA}{P} = \frac{x}{R}$

$x = R\cos\theta$

In OAP triangle,

$\sin\theta = \frac{P}{h} = \frac{AP}{OP} = \frac{OB}{OP}$

$= \frac{y}{R}$

$\Rightarrow y = R\sin\theta$

$\therefore \vec{P} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$

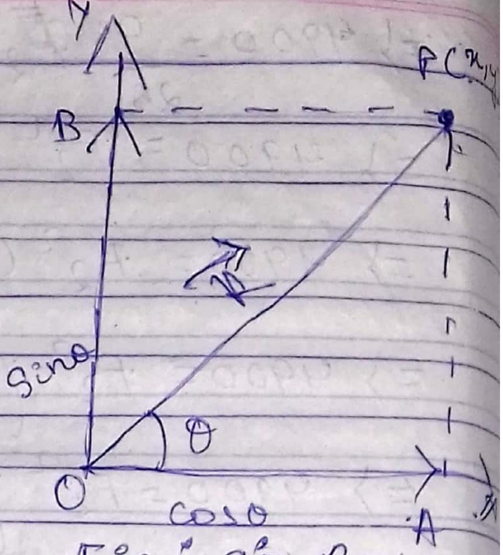

Fig:- 9:- Rectangular components of a vector.

Product of Two Vectors:-

There are two ways in which two vectors can be multiplied together.

(i) Dot product or Scalar product.

(ii) Cross product or Vector product.

(i) Dot product or Scalar product:-

Dot product of two vectors is defined as the product of their magnitudes and the cosine of the smaller angle between the two.

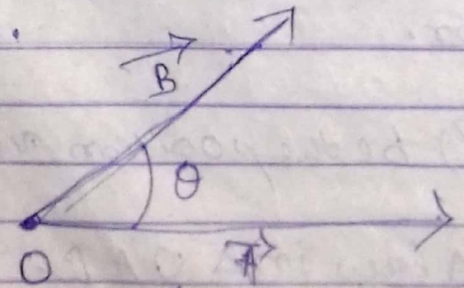

fig:- 10

Dot product of $\vec{A}$ and $\vec{B}$ is given by:-
$\vec{A}\cdot\vec{B} = AB\cos\theta$ —— (ii) where θ is the angle bet^n $\vec{A}$ & $\vec{B}$.

Characteristics of Dot product:-

(i) Commutative:- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

(ii) Distributive:- $\vec{A}\cdot(\vec{B}+\vec{C}+\vec{D}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}+\vec{A}\cdot\vec{D}$.

(iii) perpendicular vector:- The dot product of two perpendicular vectors is always zero.

Ex:- For perpendicular vectors $\vec{A}$ and $\vec{B}$
$\theta = 90^\circ$

$\vec{A}\cdot\vec{B} = AB\cos\theta = AB\cos(90^\circ)$
$= AB\times 0 = 0.$

(iv) Dot product in terms of Rectangular components:-

$\hat{i}\cdot\hat{j} = 0 = \hat{j}\cdot\hat{i}$. $\quad \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{j}\cdot\hat{k} = 0 = \hat{k}\cdot\hat{j}$.

$\hat{k}\cdot\hat{i} = 0 = \hat{i}\cdot\hat{k}$

Let A_x, A_y, A_z and B_x, B_y, B_z be the Rectangular Components of 2 vectors $\vec{A}$ & $\vec{B}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$.
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$.

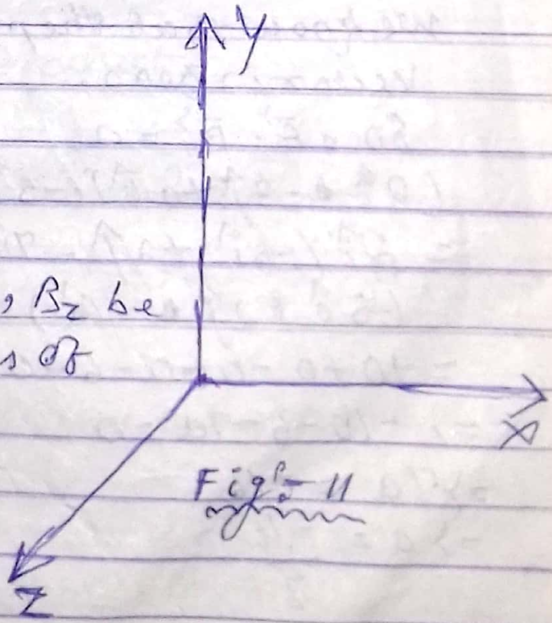

Fig:- 11

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$
$= A_x\hat{i}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= (A_x\hat{i} \cdot B_x\hat{i}) + (A_x\hat{i} \cdot B_y\hat{j}) + (A_x\hat{i} \cdot B_z\hat{k}) + (A_y\hat{j} \cdot B_x\hat{i}) + (A_y\hat{j} \cdot B_y\hat{j}) + (A_y\hat{j} \cdot B_z\hat{k}) + (A_z\hat{k} \cdot B_x\hat{i}) + (A_z\hat{k} \cdot B_y\hat{j}) + (A_z\hat{k} \cdot B_z\hat{k})$

$= A_x B_x + 0 + 0 + 0 + A_y B_y + 0 + 0 + A_z B_z$

$= A_x B_x + A_y B_y + A_z B_z$

Q. Given that $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$.
$\vec{B} = 2\hat{i} + \hat{j} - 5\hat{k}$.

Find $\vec{A} \cdot \vec{B}$

$= (\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (2\hat{i} + \hat{j} - 5\hat{k})$.

$= \hat{i}(2\hat{i} + \hat{j} - 5\hat{k}) - 2\hat{j}(2\hat{i} + \hat{j} - 5\hat{k}) - 3\hat{k}(2\hat{i} + \hat{j} - 5\hat{k})$.

$= 2 + 0 - 0 - 0 - 2 + 0 - 0 - 0 + 15$

$= 2 - 2 + 15 = 15$

Q. Given that, $\vec{A} = (2\hat{i} - 3\hat{j} + a\hat{k})$
$\vec{B} = (-5\hat{i} + 2\hat{j} - 7\hat{k})$

Find the value of a if $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

We know that the product of two perpendicular vector is zero.

So, $\vec{A} \cdot \vec{B} = 0$.

$(2\hat{i} - 3\hat{j} + a\hat{k}) \cdot (-5\hat{i} + 2\hat{j} - 7\hat{k}) = 0$

$= 2\hat{i}(-5\hat{i} + 2\hat{j} - 7\hat{k}) - 3\hat{j}(-5\hat{i} + 2\hat{j} - 7\hat{k}) + a\hat{k}(-5\hat{i} + 2\hat{j} - 7\hat{k}) = 0$.

$= -10 + 0 - 0 - 0 - 6 - 0 + 0 + 0 - 7a = 0$

$\Rightarrow -10 - 6 - 7a = 0$

$\Rightarrow 7a = -16$

$\Rightarrow a = \frac{-16}{7}$

Cross product or Vector product:-

Cross product of two vectors $\vec{A}$ and $\vec{B}$ is defined as a single vector $\vec{C}$ whose magnitude is equal to the product of their individual magnitude and the sign of the smaller angle between them and is directed along the Normal to the plane containing $\vec{A}$ & $\vec{B}$.

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} \quad \text{--- (iii)}$$

Where $\hat{n}$ is the unit vector in a direction perpendicular to the plane containing $\vec{A}$ & $\vec{B}$.

The direction of cross product can be determined by the application of following Rule.

Right Hand Thumb Rule:-

If the finger curles in the direction from $\vec{A}$ to $\vec{B}$ then the direction of thumb gives the direction of $\vec{A} \times \vec{B}$.

Characteristics of cross product:-

(i) Non-Commutative:- $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

(ii) Distributive:- $\vec{A} \times (\vec{B} + \vec{C} + \vec{D}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} + \vec{A} \times \vec{D}$.

(iii) Parallel Vector:- Cross product of two parallel vectors is always zero.

Ex:- For parallel vectors $\vec{A}$ and $\vec{B}$, $\theta = 0°$

$$\vec{A} \times \vec{B} = AB(\sin 0°)\hat{n} = 0.$$

* In case of perpendicular orthogonal unit vectors.

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$\hat{i} \times \hat{j} = \hat{k}$ $\quad$ $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\quad$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad$ $\hat{i} \times \hat{k} = -\hat{j}$

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

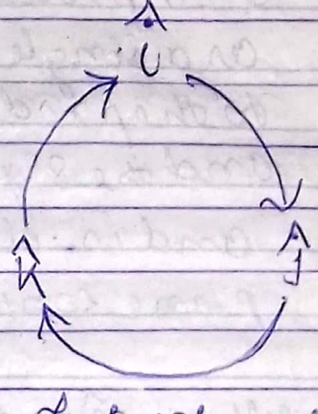

fig:- 12: cyclic Ru

(iv) Cross product in terms of Rectangular component:-

Let A_x, A_y, A_z and B_x, B_y, B_z be the Rectangular component of $\vec{A}$ & $\vec{B}$, then,

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\therefore \vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}(A_y \cdot B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) +$$

$$\hat{k}(A_x B_y - A_y B_x).$$

problem:- 4:- $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$

Find $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -2 & 3 \end{vmatrix}$

$\Rightarrow \hat{i}(9-(-8)) - \hat{j}(6-4) + k(-4-3)$

$\Rightarrow 17\hat{i} - 2\hat{j} - 7\hat{k}.$